Statistical Modeling the relationship between pension funds, inflation in Chile and its incidence on the interest rate under a COVID-19 perspective

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I dedicate this dissertation to my wife, Ingrid Almonacid Sepúlveda, who has supported me throughout the process. Special thanks to my tutor, Professor Dr. Milan Stehlík, for their support, and encouragement.

"What is essential is invisible to the eyes."

The Little Prince

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Publications

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5. Aravena-Gaete, M., Flores-Noya, D., Ruete, D., & Leal, D. (2021). **Evaluative methodology to develop higher skills** In T. V. Petkova & V. S. Chukov (Eds.), 7th International e-Conference on Studies in Humanities and Social Sciences: Conference Proceedings (pp. 15-26). 28 June 2021, Belgrade: Center for Open Access in Science. https://doi.org/10.32591/coas.e-conf.07.03015a

Book Chapter

1. **Argumentación y habilidades de pensamiento en la formación del profesorado competencias pedagógicas para la educación en contextos de incertidumbre**. Andrea Figueroa Vargas, Danilo Leal Moraga, Trinidad Zúñiga Celis. Desafíos de investigación educativa durante la pandemia COVID 19 / coord. por José María Romero Rodríguez, Inmaculada Aznar Díaz, Antonio José Moreno Guerrero, Carmen Rodríguez Jiménez, 2022, ISBN 978-84-1122-034-7, págs. 156-165.

2. **La literatura como didáctica para el desarrollo de habilidades cognitivas**. Trinidad Zúñiga Celis, Kissy Aravena Gaete y Danilo Leal Moraga. Estrategias para desarrollar habilidades del pensamiento en la educación superior: variables ocultas y estrategias. Margarita Aravena Gaete, David Ruete Zúñiga, Natalia Campos Soto, Andrea Figueroa Vargas. ISBN: 978-956-6095-76-7. DOI: https://doi.org/10.26448/ae.9789566095767.62

Talks and Poster Conferences

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2. 2021. Presentation at XV Congreso Internacional de Mercados **“Spatial analysis, fundamentals and the importance for marketing.”**, organized by Universidad Libre Seccional Socorro, Colombia y Universidad Andres Bello, Chile.

3. 2020. Poster Presentation at XLVIII Coloquio Argentino de Estadística. **“Proposal of a Bayesian Space-Time Model for the Estimation of Drinking Water Consumption in Chile.”** Calderón Daniel, Leal Danilo, Nicolis Orietta. https://rdu.unc.edu.ar/handle/11086/16914

4. 2019. Poster Presentation at National Research Conference at Andrés Bello University. **"Spatial Econometric: Modeling of Crime in Santiago of Chile.”** Leal Danilo.

5. 2017. Poster Presentation at National Statistical Conference. **“Robust volatility estimation in the Chilean stock market”** Leal Danilo, Maidana Jean-Paul.

Abstract

Spanish Version

Esta tesis profundiza en dos aspectos fundamentales de la dinámica de los mercados financieros: la calibración de modelos de tasas de interés y la respuesta de los mercados bursátiles a eventos comerciales internacionales significativos. La primera parte del estudio se centra en el mercado bursátil chileno, en particular el IPSA, entre 2016 y 2019, un período marcado por eventos comerciales cruciales entre Estados Unidos y China. A través del análisis de los Retornos Anormales Acumulados (CAR), el estudio descubre una reacción mixta del IPSA, con una notable pérdida de capitalización bursátil. Esta investigación pone de relieve la no normalidad en la distribución de los rendimientos esperados, marcada por una significativa asimetría y leptokurtosis. También enfatiza la importancia de la modelación estadística para entender y cuantificar los cambios en los volúmenes de comercio, especialmente en el contexto de la guerra comercial entre Estados Unidos y China y su impacto en el mercado bursátil chileno. La segunda parte del estudio destaca la importancia de clasificar los datos de las series temporales financieras en clases homogéneas para calibrar eficazmente los modelos de tipos de interés. La estrategia propuesta en este estudio consiste en utilizar umbrales de p-valores para comprobar la normalidad. Este método se aplica a las series financieras del IPSA como referencia. Uno de los hallazgos más intrigantes de este enfoque es la correlación positiva observada entre los tasas de interés y los rendimientos del mercado, como indica el IPSA en determinados rezagos. Este descubrimiento presenta una notable desviación de las teorías financieras convencionales, desafiando las creencias establecidas y ofreciendo nuevas perspectivas sobre la dinámica de los mercados financieros. También empezamos a investigar el impacto de la pandemia de COVID-19 en el sistema de pensiones chileno. Se aborda la respuesta única de Chile, uno de los tres únicos países en todo el mundo, para permitir retiros de los fondos de pensiones obligatorios como estrategia de mitigación de la crisis. El estudio examina la aplicación y las consecuencias de tres retiros autorizados por el gobierno, revelando el retiro por parte de los ciudadanos chilenos de aproximadamente

50.334 millones de USD. Este importante movimiento financiero tuvo implicancias de gran alcance, como dejar a 3,8 millones de personas sin sus ahorros de pensiones.

English Version

This Thesis delves into two pivotal aspects of financial market dynamics: the calibration of interest rate models and the response of stock markets to significant international trade events. The first part of the study focuses on the Chilean stock market, particularly the IPSA, from 2016 to 2019, a period marked by crucial trade events between the U.S. and China. Through the analysis of Cumulative Abnormal Returns (CAR), the study uncovers a mixed reaction of the IPSA, with a notable loss in market capitalization. This research highlights the non-normality in the expected return distribution, marked by significant asymmetry and leptokurtosis. It also emphasizes the importance of statistical modeling in understanding and quantifying the changes in trade volumes, especially within the context of the U.S.-China trade war and its impact on the Chilean stock market. The second part of the study underscores the importance of classifying financial time series data into homogeneous classes for effective calibration of interest rate models. The proposed strategy in this study involves using p-value thresholds to test for normality. This method is applied to the financial series of the IPSA as a benchmark. One of the most intriguing findings from this approach is the observed positive correlation between interest rates and market returns, as indicated by the IPSA in specific lags. This discovery presents a noteworthy deviation from conventional financial theories, challenging established beliefs and offering new insights into the dynamics of financial markets. We also started to investigate the impact of the COVID-19 pandemic on the Chilean pension system. It addresses the unique response of Chile, one of only three countries worldwide, to permit withdrawals from mandatory pension funds as a crisis mitigation strategy. The study examines the implementation and consequences of three government-sanctioned withdrawals, revealing Chilean citizens' total withdrawal of approximately 50.334 billion USD. This significant financial movement had far-reaching implications, including leaving 3.8 million people with depleted pension savings.

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Chapter 1

Introduction and work summary

1.1 General overview

Chile is a country with a small population but has a fairly developed market compared to other Latin American countries. It includes stock brokers, securities agents, banks, three stock exchanges, insurance companies, mutual funds, domestic and foreign investment funds, and pension funds.

Interest rates are one of the most important financial instruments in an economy as they are a crucial monetary policy tool for the central banks of various countries. Having low-interest rates encourages consumption and, on the contrary, having high-interest rates encourages saving. Through the Monetary Policy Rate, central banks determine the range of interest rates, in order to control inflation, among other things.

In recent years as a result of COVID-19, all markets have presented certain anomalies in interest rates. Some countries have very low or even negative interest rates like Chile, having high interest rates with the intention of controlling inflation. For comparison with effects on selected aspects of US economy, see Kuenstler et al. (2023), where EUR/USD exchange rates are discussed in period 13/10/2019-9/4/2020.

Ignacio Barra Novoa (2021) provides an initial analysis of the impact of the COVID-19 pandemic on the macro and microeconomic environment of Chile. Ignacio Barra Novoa (2021) finds that the number of active firms has decreased significantly, partly due to the social crisis that started on October 18, 2019. The pandemic has further impacted most industries, resulting in job losses and reduced business profitability. The implications of these findings are significant for the central bank's policies. They can help predict medium and long-term projections, particularly for the economic and social growth of the country.

From a labor point of view Rivera and Castro (2021) underscores the challenges faced due to high levels of labor informality and dependency on daily income, compounded by insufficient state support. Using longitudinal data, the study reveals a gender disparity in employment impacts: men faced less job loss. They re-entered the labor market more quickly than women, who often transitioned from paid employment to unpaid care work. This shift is indicative of deeper gender inequality issues exacerbated during economic crises. The research also finds a strong correlation between labor informality, low-skilled jobs, and higher unemployment rates in Chile, where a considerable segment of the population works under such conditions. The study concludes by emphasizing the importance of a gender-sensitive approach in current crisis management and future economic recovery plans.

According to the research by Madeira (2023), a simulation was done to analyze the effect of Chile's pandemic policies on household consumption. The study discovered that if public transfers and a quarantine flexibilization policy were not implemented, household consumption would have decreased by 16.7%. Even with only a quarantine flexibilization policy in place, consumption would have still decreased by 10.2%, which highlights the significant role of public transfers. By implementing both measures, household consumption remained 6.2% less than pre-pandemic levels.

The preceding makes it essential to develop flexible models that can predict various economic variables accurately. This is necessary to anticipate and comprehend the impacts of catastrophic events or crises. Such an understanding can lead to sustainable economic growth in line with the United Nations guidelines.

For this reason, we decided to study the behavior of different economic variables that would allow us to approach the problem comprehensively, including the particularity of Chile and the economic-financial decisions that were taken as a result of COVID-19. One might think that such events are isolated and will not be repeated much in the future. However, events such as the war between Russia and Ukraine and, more recently, the war between Israel and Palestine have caused high volatility in international markets.

This thesis comprises two papers, which correspond to each of the following chapters of this document. First, we have the article **"Stochastic approach to heterogeneity in short-time announcement effects on the Chilean stock market indexes within 2016-2019"**

(Chapter 2). The study assesses the impact of U.S.-China trade war events (2016-2019) on the Chilean stock market index (Indice de Precio Selectivo de Acciones, IPSA). It examines how these international events influenced local equity markets, assuming that global economic actions, especially involving major economies like the U.S. and China, have significant implications for smaller, third-party economies like Chile.

The study adopted the Mean Adjusted Model Method for event studies, targeting 26 significant trade war events. It utilized daily closing values of IPSA, measuring Cumulative Abnormal Returns (CAR) for each event. Where AR_{it} = abnormal return i on day t of the event window; $\mathrm{AR}_{it} = R_{it} - E[R_{it}]$, and CAR_t = cumulative abnormal return for the event is $\mathrm{CAR}_t = \sum_{i=1}^{n} \mathrm{AR}_{it}$. This involved calculating expected returns $(E[R])$ for an 8-day event window (2 days before, the event day, and five days after), employing 60 days of prior data to establish a pre-event average. Statistical analysis was conducted to determine abnormal returns, cumulative abnormal returns, and average cumulative abnormal returns, with a paired t-test to assess if the changes were statistically significant.

The model was fitted using experimental data and software tools like Maple 2019. Parameters such as a, b, c, and e were estimated for two different periods, indicating distinct market behaviors before and after a threshold time-point, which was identified as being influenced by the COVID-19 pandemic.

The study used data-driven confidence intervals and the delta method to create confidence intervals for the aggregated variance of the stock market index. This approach allowed for comparing market behavior before and after the identified change point.

Robust normality tests (Anderson-Darling, Jarque-Bera, Lilliefors, Shapiro–Wilk, and others) were applied to the data sets. These tests indicated significant deviations from the normal distribution, mainly due to outliers in the expected returns

Main results:

- CAR analysis: out of 26 events, 16 showed negative market reactions, and ten were positive. The Chilean market experienced a net negative impact with an aggregate value decline of -10.04
- Market capitalization loss: the cumulative effect on market capitalization was a net loss of approximately 13 billion USD.

- Statistical significance: the paired t-test indicated that 18 out of the 26 events led to statistically significant reactions in the IPSA index. Among these, 11 events caused a negative change, and seven led to positive reactions, confirming a predominantly negative effect of the U.S.-China Trade War on the IPSA.
- Market behavior: the stochastic model revealed different statistical behaviors of the IPSA before and after the identified threshold, suggesting significant changes in trade volumes and market dynamics.
- Economic implications: the study underlines that the impact of events like the U.S.-China trade war on stock markets, such as Chile's IPSA, is variable and can be influenced by broader economic cycles and financial system instabilities. It suggests that global financial crises and protectionist measures like trade wars are interconnected and can have far-reaching impacts beyond the primary economies involved.

In the second article, **"On testing the changes in trends of IPSA and rates"** (Chapter 3), We address the challenge of calibrating interest rate models by grouping data into homogeneous classes. This issue is crucial in financial time series analysis, particularly for more realistic interest rate models beyond the Cox-Ingersoll-Ross (CIR) model. The focus is on developing a strategy for general class interest rate models where classes are formed based on p-value thresholds for testing for normality and gamma distributions. A key aspect studied is the relationship between interest rate and market returns, specifically using the IPSA (Index of Selective Stock Prices) as the benchmark series.

The methodology involves transforming financial data into approximately independent sub-groups. The approach recalls tests for homogeneity of exponential distribution and robust tests for normality, allowing for the classification of subclasses. The article references a likelihood ratio-based test for subgrouped data, known as the exact likelihood ratio test for homogeneity (ELR). It assumes that each observation follows an exponential distribution with its parameter. This model has a submodel of non-homogeneity with unobserved clustering and a given number of clusters.

This article presents two-fold results based on simulation and by usage a new testing model that focuses on constructing empirical subgroups and parameter dependence models. A significant finding from applying this model to the Index of Selective Stock Prices (IPSA) data is the observed positive correlation between interest rates and stock market returns, at least in several lags. This outcome contradicts the conventional understanding of finance, which suggests that higher interest rates should negatively impact stock market prices.

Main results:

- Interest rate and stock market correlation: the study finds, for a certain lags, a positive correlation between interest rates and stock market returns, indicating that higher interest rates are associated with higher returns in the stock market. This discovery challenges the traditional belief that rising interest rates make stock investments less attractive than interest-bearing bonds or other debt securities.
- Long-term correlation dynamics: the conventional wisdom holds true only in the very short term. In the medium to long run, higher interest rates contribute to faster-growing deposits, integrating into the economy's broader money supply (like cash, demand deposits, time-related deposits, saving deposits, and other larger liquid assets). This growth in money supply leads to more investments in markets such as stocks and real estate.
- Impact on asset prices: the study suggests that the faster money supply growth, partly due to banks granting more loans to pay interest on growing deposits, increases the money supply further through the "money-multiplier" effect. This phenomenon potentially inflates asset prices, including those in the stock market.

1.2 Interest rate modelling

The most widely used approach has been to assume the short-term interest rate follows a diffusion process, see Svoboda (2004), and Sam (2004) a Markovian process for which all realizations or sample functions $\{X_t, t \geq 0\}$ are continuous. A summary of the main models used is presented below from Svoboda (2004).

Two models are considered relevant for interest rate modeling. The first is the mathematical method of modeling interest rate movements based on market risk, time, and long-term equilibrium interest rate values, introduced by Vasicek (1977). The second is the CIR model by Cox and Ross (1985), a complete equilibrium model. In this model, bond prices are derived from exogenous specifications of the economy, such as production opportunities, investors' tastes, and beliefs about future states of the world.

Multiple academic studies have proposed different models to understand the behavior and dynamics of interest rates. Brennan and Schwartz (1979) proposed a model that assumes the entire term structure can be expressed as a function of the yields of the longest and shortest maturity default-free bonds. Longstaff and Schwartz (1992) developed a two-factor model of the term structure based on the framework of Cox and Ross (1985). The two factors are the short-term interest rate and the instantaneous variance of changes in this short-term interest rate (volatility of the short-term interest rate). This model suggests that the prices of contingent claims reflect the current interest rate levels and their volatility. Additionally, Langetieg (1980) developed a general framework where the short-term interest rate is expressed as the sum of a number of underlying stochastic factors. This model is essentially an extension of Vasicek's approach, where the evolution of the short-term interest rate is subject to multiple sources of uncertainty. In their proposal for preference-free pricing, Ball and Torous (1983) suggest using the bond as the underlying state variable. They assume that the price of a risk-free zero coupon bond follows a Brownian bridge process. This ensures that the bond price converges to its face value at maturity. The process of fitting the initial term structure can be made more accessible by using an extension proposed by Hull and White (1996) to the Vašíček and CIR models. This extension allows for time-dependent drift and volatility parameters, resulting in analytical solutions for bonds and bond options. By determining model parameters, including those related to the market price of risk, in terms of the initial term structure, an exact fit can be achieved for the initial term structure of interest rates and possibly interest rate volatility. In their study, Ho and Lee (1986) proposed a new method for modeling the term structure. Rather than focusing on the short-term interest rate, they developed a discrete-time model considering the entire yield curve's evolution. In one-factor models, the short-term interest rate is usually the only factor determining the entire yield curve. However, the Ho and Lee model allows for flexible initial yield curve specifications, making it possible to calibrate the model to the observed initial yield curve.

1.2.1 Kiguradze class interest rate

This sub-section presents a stochastic interest rate model that has the particularity of capturing long run trends.

The independent variable will be damping parameter m of the general form of 2nd order interest rate model which is a generalization of standard interest rate models like CIR or Vašíček. By general form of 2nd order interest rate model we understood as arbitrary solution of (two dimensional) system of stochastic differential equation (see Stehlík et al. (2017))

$$\mathrm{d}\mathbf{X}_t = \tilde{\mathbf{b}}(t,\mathbf{X}_t)\mathrm{d}t + \Sigma(t,\mathbf{X}_t)\mathrm{d}\mathbf{W}_t, \tag{1.1}$$

for $t \in I = [0, \infty)$ where $\mathbf{X}_t = (r_t, p_t)^\intercal$, r_t is interest rate and p_t represents generalized force of interest, and

$$\tilde{\mathbf{b}}(t, \mathbf{X}_t) = \left(c(t)(p_t)^m, a(t)(p_t)^l + b(t)(r_t)^n\right)^T, \Sigma(t, \mathbf{X}_t) = \left(\begin{smallmatrix} 0 & 0 \\ 0 & \sigma(t)(r_t)^k \end{smallmatrix}\right)$$

where $a, b, c \in C(I),\ c \neq 0,\ \sigma \geq 0$ and $n, k, l \in \mathbb{Q}$ and $m \in \mathbb{Q} \setminus \{0\}$ with odd denominators.

Specifically, we assume here the system

$$\begin{aligned} \mathrm{d}r_t &= c\,(p_t)^m\,\mathrm{d}t \\ \mathrm{d}p_t &= [a\,p_t + b]\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \end{aligned} \tag{1.2}$$

Here $(p_t)^m$ can represents either a normal power function or the so-called signed power function

$$\Phi_m(z) = |z|^{m-1} z,\ m > 0, \tag{1.3}$$

which is used to avoid problems with the definition of domain of a power function. For such a diffusion processes the parameter m is some kind of stabilization parameter and the careful financial management requires checking of m.

Trajectories for the specific values $m = 1, 3$ and development of United Kingdom interest rate from October 2019 to August 2020 has been studied in Stehlík et al. (2020).

Let us consider for the sake of simplicity the isotropic stationary process

$$m(t) = \theta + \varepsilon(t) \tag{1.4}$$

with the times as design points $t_1, \ldots, t_n$ taken from a compact design space X, $m(t)$ denotes measurement of damping variable at time point t and $\varepsilon(t)$ is error at point t. For the sake of simplicity we consider $X = [a, b]$. The trend parameter $E(m(t)) := \theta \in \Theta$ is unknown, the covariance function $C(d)$ at lag d depends on another unknown covariance parameter $r \in \Omega$ and d_i is the lag between two particular design points t_i and t_{i+1}. Parametric spaces Θ and Ω are open sets with respect to standard topology.

We assume the class of positive definite measurable functions $C_r(d) : \Omega \times \mathbb{R}^+ \to \mathbb{R}$ such that

- $C_r(0) = 1$ for all $r \in \Omega$

- for all r mapping $d \to C_r(d)$ is semicontinuous
- $\lim_{d\to+\infty} C_r(d) = 0$.

Alternatively, we say that C_r is from abc+ class, if it is abc and moreover $C_r(d) \geq 0$, see Stehlík et al. (2017).

1.2.2 IPSA and Interest Rate

The IPSA represents the return of the market as well as other stock market indicators in different stock exchanges around the world, e.g., Ibex 35 in Spain, S&P 500 in the United States, and Bovespa for Brazil, among others. On the other hand, the interest rate represents the return obtained by a financial institution when lending money, and from the consumer's point of view, it is the cost of being able to use the money. The relationship between interest rates and the market return has been studied in different countries, and the result depends on the country analyzed; you can see (Canova (1997), Lee (1992), Øystein Gjerde and Sættem (1999) and references therein). For our case, we realize a cross-correlation analysis between current interest rate and IPSA from 2016 to 2022.

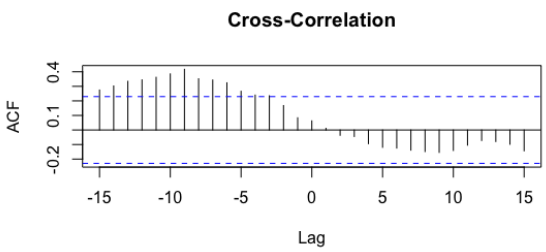


Fig. 1.1 Cross-Correlation (CC).

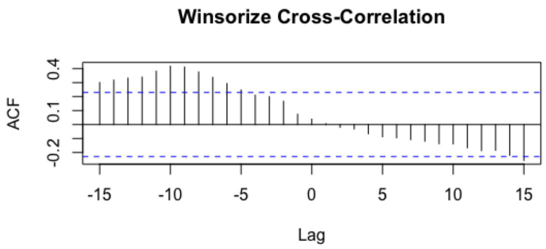


Fig. 1.2 Winzorize Cross-Correlation (WCC).

Figures 1.1 and 1.2 represents the correlation between IPSA and Interest rates at different lags, and Table 1.1 shows the cross-correlations values. We also calculate a robust estimation using R-package datawizard described in Patil et al. (2022). Robustness to outliers is used, using the percentile method with a threshold of 0.2

Lag	CC	WCC	Lag	CC	WCC
0	0.063	0.038	0	0.063	0.038
-1	0.084	0.074	1	0.011	0.006
-2	0.168	0.167	2	-0.036	-0.020
-3	0.235	0.199	3	-0.045	-0.031
-4	0.240	0.210	4	-0.094	-0.066
-5	0.266	0.247	5	-0.119	-0.087
-6	0.324	0.293	6	-0.124	-0.095
-7	0.343	0.338	7	-0.138	-0.108
-8	0.352	0.376	8	-0.148	-0.119
-9	0.417	0.412	9	-0.154	-0.137
-10	0.385	0.417	10	-0.141	-0.138
-11	0.362	0.382	11	-0.104	-0.166
-12	0.344	0.339	12	-0.072	-0.187
-13	0.334	0.332	13	-0.079	-0.184
-14	0.303	0.318	14	-0.098	-0.220
-15	0.275	0.301	15	-0.143	-0.257

Table 1.1 Auto-correlations of series by lag.

1.3 Objectives

Due to various shocks around the world, such as the pandemic, the war between Russia and Ukraine, and the recent war between Palestine and Israel, economic variables are becoming increasingly complex and volatile. With the interconnectedness of global economies, it is essential to study and develop new models to help us understand these interactions. By doing so, we can analyze how to assist individuals and companies in navigating such complex economic situations. Based on the preceding, the following objectives are formulated.

GENERAL OBJECTIVE

Propose a novel modeling for the interest rate structure in Chile under a COVID-19 perspective.

SPECIFIC OBJECTIVES

1. SPECIFIC OBJECTIVE 1. Analyze the principal variables behavior in the structure of the interest rates before and during COVID-19.

   The first paper provides a detailed analysis of stock market returns during significant economic events. This analysis can be used to understand the impact of these events on interest rates. The second paper offers a more direct analysis of interest rate models and their relationship with market returns. This includes a study of the period that covers the COVID-19 pandemic. Both documents together can provide valuable insights into the behavior of key variables in interest rate structures before and during the COVID-19 pandemic.

2. SPECIFIC OBJECTIVE 2. Analyze changes in the pension system in Chile both from qualitative and quantitative perspectives.

   The financial models and statistical techniques used in these two presented papers could be adapted or used as a foundation for analyzing pension system data. A comprehensive study of the pension system requires additional data sources. We have started working on it and you can see our progress in 1.4.

   An interesting analysis is presented by Madeira (2022), and it will be a key research for our third article. The study analyzes Chilean policy changes (2019-2022) and their impact on household savings and pensions. It finds that households spend much of their public pension, leading to lower savings rates. COVID-19 pension withdrawals have further reduced savings. Proposed reforms, such as higher contributions and delayed retirement, could increase savings, but allocating more to solidarity funds might reduce them. Delaying retirement boosts pension replacement ratios significantly, but solidarity schemes, while benefiting low-income households, could decrease high-income retirees' pensions by 2055.

3. SPECIFIC OBJECTIVE 3. Based on results of SPECIFIC OBJECTIVES 1 & 2 propose a flexible model to predict low/high-interest rates and behavior of parameter m in pandemic year.

   The methodologies and models in both papers could serve as a basis for developing a predictive model for interest rates. The insights gained from analyzing interest rate trends and financial time series before and during COVID-19 can inform the development of a model that accounts for pandemic-related volatility. The behavior of parameter m, which might be specific to a particular model or context, could be analyzed based on the statistical and econometric methods outlined in these documents.

1.4 Pension Funds: The Chilean case in times of pandemic

As is known worldwide, the pandemic produced by COVID-19 has had very negative consequences in different areas of our lives, including pension funds. That is why we are working on a third paper on the subject, which was not fully prepared for this thesis work, but the progress to date is presented.

The International Federation of Pension Fund Administrators (FIAP (2021) in Spanish) published a study in January of this year in which it analyzes the impact of the pandemic on employment, the number of contributors, and the collection of pension funds in different countries of the world, indicating that only three countries in the world (Australia, Chile and Peru) allowed withdrawals from mandatory pension funds, while other countries allowed withdrawals from voluntary savings accounts, as a measure adopted by governments to help them cope with the crisis.

In the case of Chile, three withdrawals from mandatory pension funds were authorized, which could be total for those with a balance of less than 35 UF (approx. 1,346 USD) or 10% with a cap of 150 UF (approx. 5,767 USD), with only the second withdrawal being subject to tax for those with a monthly income of more than CLP 1,500,000 (approx. 1,890 USD). These withdrawals were approved by three constitutional reforms at different times.

Withdrawal	Law	Date of promulgation	Number of Payments	Withdrawn MM USD
1	N° 21.248	July 2020	10.820.335	$ 20.923
2	N° 21.295	December 2020	8.373.941	$ 16.721
3	Nº 21.330	April 2021	6.333.531	$ 12.690

Table 1.2 Number of payments made to individuals and total amounts withdrawn as by June 25, 2021

As can be seen in Table 1.2, the total withdrawal allowed amounts to 50,334 million dollars (MM USD) in less than 12 months, reaching a total of 25,527,807 withdrawals by the inhabitants of Chile. The Superintendence (2021) of pension, in its analysis, shows that due to these withdrawals, 3.8 million of people have been left without balances in their pension savings accounts, and also that a gender gap has been evidenced since women withdraw a smaller amount of resources, which also represents a higher percentage of their balance.

Finally, the same institution concludes that withdrawals represent an average loss of 33.1% of pension benefits for women and 24.1% for men. This is equivalent to losing, on average, 6.3 years of contributions for women and 5.6 years for men.

The Financial Stability Board (CEF (2021) in Spanish), in its analysis of pension fund withdrawals, points out that they threaten financial stability in the following ways:

1. The accelerated liquidation of investments in financial instruments in Chile and the world impacts the prices of such instruments in the local market and the exchange rate.
2. Due to uncertainty in the financial market, there are expectations of future withdrawals.
3. Higher fiscal financing needs resulted from increased spending in the solidarity pension system and reduced revenue from tax exemptions.
4. Reduced domestic savings and a shallow capital market have long-term negative effects on investment, financing costs, and economic growth.

On the other hand, the president of the Central Bank (Banco central de chile (2023)) spoke before the Economy Commission of the Chamber of Deputies, stating that "pension fund withdrawals have only had an impact on inflation as a result of the higher consumption financed with the withdrawals, but not because of the operations of the Issuing Institute" concerning the possible impact of bond purchases on the money supply and inflation.

Undoubtedly, the withdrawals from the pension funds are detrimental to the retirement of hundreds of people in Chile, proving once again that people prefer current consumption to future consumption.

Whether we like it or not, there is inflation in Chile and the rest of the world, which is partly explained by external (see Banco central de chile (2023) section: Politics) factors but also by internal factors such as the withdrawal of pensions, which resulted in the increase of the monetary policy rate by the central bank to try to counteract inflation, but strongly punishing bank loans, especially mortgage loans.

The Pension Fund Administrators (AFP in Spanish) currently manage five pension funds, which vary from the riskiest (Fund A) to the most conservative (Fund E), and the contributor is free to choose which of these funds he wants to make his savings profitable.

Figure (1.3) shows the average annual returns from 2019 to 2022, in which a slight difference can be seen between Fund A and Fund E.

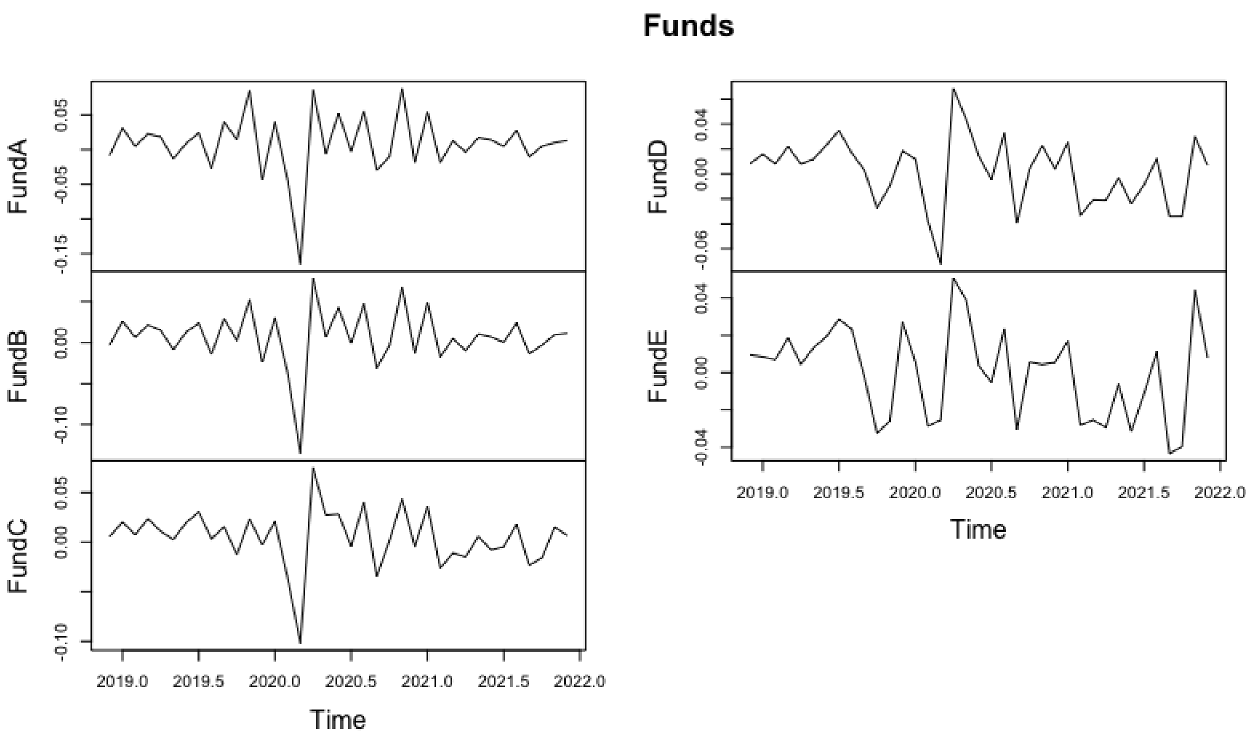


Fig. 1.3 Evolution of pension funds from 2019 to 2022

The profitability of the pension funds is related to the type of instrument in which the pension fund manager invests, which is classified into fixed-income and variable-income instruments. Fixed income is understood as those investment instruments that deliver a known return in a determined period, considered low risk since they are not subject to large market volatilities. On the other hand, variable income instruments are representative of the property or capital of a company or enterprise. Those who buy these shares become owners or shareholders and, therefore, will obtain profits or losses depending on how the company is doing, i.e., they have higher volatility of expected returns and are therefore considered riskier.

The composition of the pension funds is shaped according to the percentage of resources to be invested in variable income instruments regulated by the state. Table 1.5 shows that Fund A can invest more in variable income instruments (80%). At the same time, Fund D is mainly composed of fixed income, which directly influences the profitability of each contributor.

Found	Max	Min
Fund A	80%	40%
Fund B	60%	25%
Fund C	40%	15%
Fund D	20%	5%
Fund E	5%	.

Table 1.3 Equity Investment Limits by funds.

Both financial theory and empirical evidence have shown us that long-term returns always have an increasing trend. Given the above, the AFPs recommend investing in more volatile funds for young people with a longer savings horizon and, consequently, a recovery from possible drops in returns that may be generated. On the other hand, for those close to retirement age, it is recommended to invest in funds that are as less volatile as possible since they do not have the same savings horizon.

However, particular events like: the pandemic, financial crises, natural disasters, wars, among others cause the profitability to be lower or even negative, which generates a consequence in the final profitability to be obtained, therefore if there were an instrument that would allow anticipating such events, it would be possible to obtain a higher future profitability.

Consequently, would be necessary to have a tool capable of anticipating/understanding the behavior of profitability to optimize each contributor's funds so that they can change their savings funds and thus optimize their retirement. The ability to predict the performance of pension funds will allow contributors to switch funds in advance and thus protect themselves from possible losses.

As stated in specific objective 2, the main variables that could significantly affect the behavior of the pension funds are studied; among them are the average interest rate, the market return (IPSA), and a geometric series (m) that models increasing dumping in the period from June 2019 to June 2021 is considered, see equation 1.4

Date	m	IPSA	Interest Rate	Pension Return
June 2019	1.000.000	5070.72	0.02030	0.01662
July 2019	1.010.000	4972.36	0.02460	0.02820
August 2019	1.020.100	4804.37	0.01900	0.00042
September 2019	1.030.301	5059.04	0.01720	0.01706
October 2019	1.040.604	4744.13	0.01790	-0.01118
November 2019	1.051.010	4538.8	0.01800	0.02512
December 2019	1.061.520	4669.85	0.01800	-0.00476
January 2020	1.072.135	4572.06	0.01910	0.02188
February 2020	1.082.857	4122.63	0.01860	-0.03700
March 2020	1.093.685	3487.49	0.01850	-0.09996
April 2020	1.104.622	3977.56	0.02210	0.07180
May 2020	1.115.668	3647.6	0.02440	0.02216
June 2020	1.126.825	3959.02	0.02330	0.02830
July 2020	1.138.093	4017.08	0.02400	-0.00352
August 2020	1.149.474	3767.15	0.02150	0.03988
September 2020	1.160.969	3637.3	0.01800	-0.03324
October 2020	1.172.579	3539.92	0.02140	-0.00052
November 2020	1.184.304	4032.87	0.02070	0.04518
December 2020	1.196.147	4177.22	0.02000	-0.00540
January 2021	1.208.109	4288.65	0.01890	0.03620
February 2021	1.220.190	4573.37	0.02080	-0.02488
March 2021	1.232.392	4898.09	0.02100	-0.00794
April 2021	1.244.716	4472.84	0.02010	-0.01576
May 2021	1.257.163	4356.43	0.02040	0.00488
June 2021	1.269.735	4331.33	0.02170	-0.00850

Table 1.4 Observed economics variables for the period june 2019 to june 2021 Source: Banco central de chile (2023).

$$Call : lm(formula = m \sim IPSA + InterestRate + PensionReturn)$$

Coefficients	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1184692	52734	22.465	3.63e-16 ***
IPSA	-3501	2184	-1.603	0.124
Interest Rate	2834	2254	1.257	0.222
Pension Return	-3370	2149	-1.568	0.132

Table 1.5 Geometric series (m) models increasing lineal dumping. IPSA, interest rate and pension return are independent variables in the linear model.

```
Signif. codes:  0   ***   0.001   **   0.01   *   0.05   .
   0.1          1\\
Residual standard error: 76670 on 21 degrees of freedom\\
Multiple R-squared:  0.2478,    Adjusted R-squared:  0.1403\\
F-statistic: 2.306 on 3 and 21 DF,  p-value: 0.1061\\
```

We calculate Cook's distance to identify influential points. Observation numbers 2 and 20 stand out.

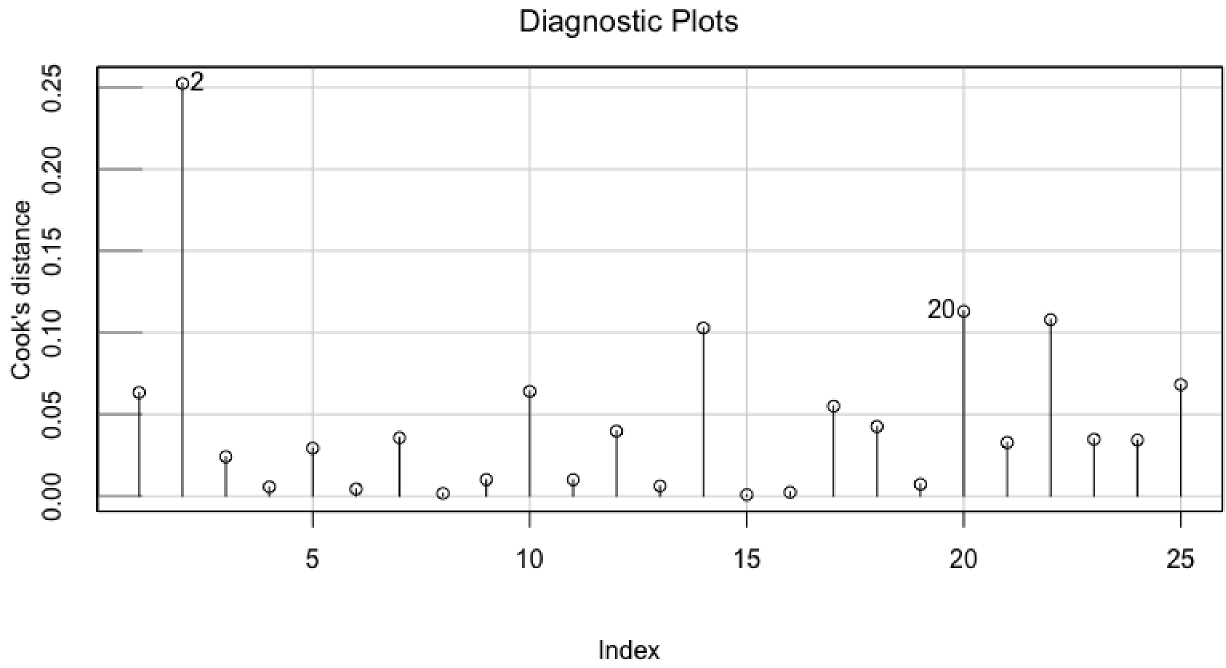


Fig. 1.4 Cook's distance.

Upon reviewing the residuals analysis (see Figure 1.4), we found that there are several large residual values, namely 2, 20, 23, and 24.

Observation 20 corresponds to January 2021. In the following months, there was high volatility in average pension fund yields and an increase in market yields.

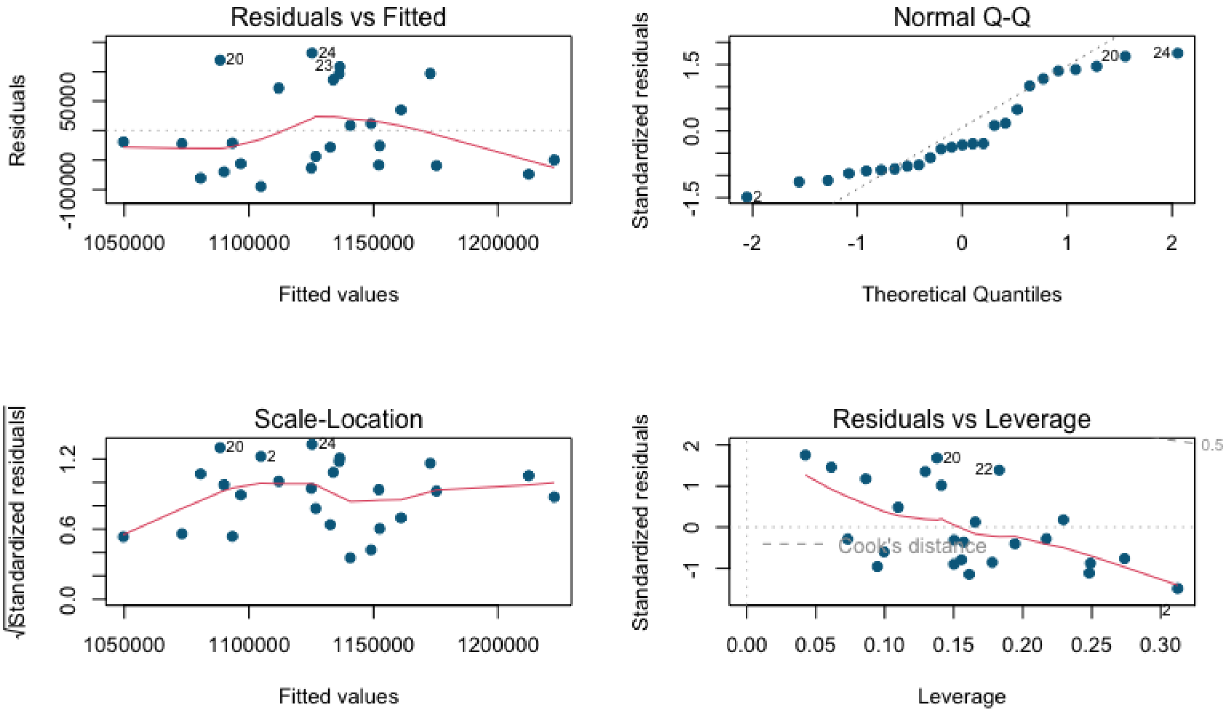


Fig. 1.5 Residual Analysis.

1.4.1 Monetary Policy Rate

For a better understanding of the financial movements that occurred during the pandemic, it is essential to know the economic measures taken by the countries, and in particular, we can find the economic measures of Chile in InvestChile (2023). On the other hand, we can find measures taken by the Central Bank of Chile during COVID-19 see (Garcia (2021)). Within the most important one, we can find that it decided to lower the monetary policy rate to its effective lower limit (ELB, evaluated at 0.5% per year) to have liquidity in the market.

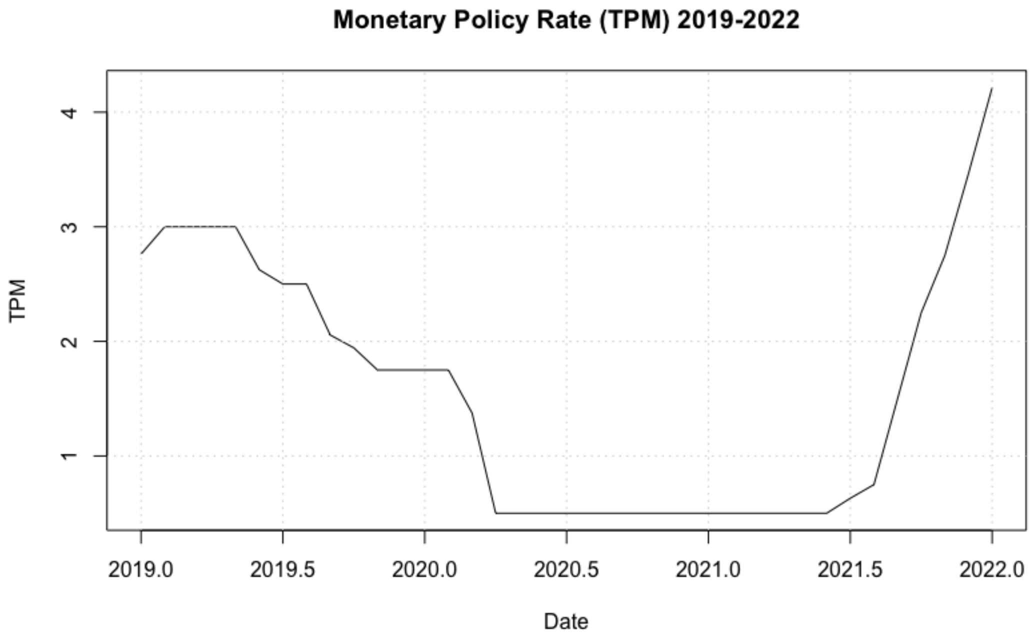
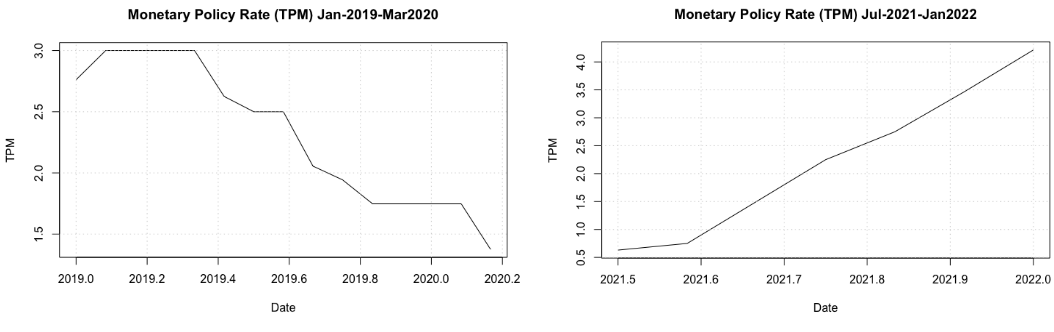


Fig. 1.6 Monetary Policy Rate (TPM in Spanish).

As we can appreciate, Figure 1.6 represents the monetary policy rate set by the central bank of Chile between January 2019 and January 2022. This rate was defined to address the various economic situations that arose during that period. From April 2020 to June 2021, the monetary policy rate remained steady at 0.5% to combat the pandemic.



(a) TPM before. (b) TPM after.

Fig. 1.7 Before and after constant rate.

To perform better analysis, we divided the time series into a before and after constant TPM as shown in Figure 1.7 and then performed the normality (Shapiro-Wilk), (Lilliefors), and exponential (Moran) tests. As we can see in Table 1.6, the Shapiro-Wilk test indicates that only the after windows present normality on the data with a p-value of 0.7155. For the

Lilliefors test, normality is presented in both cases before and after windows with a p-value of 0,2717 and 0,9177, respectively. In the exponential Moran test, just in the case of the after window, we can find exponentiality with a p-value of 0,2193.

Test	Shapiro-Wilk		Lilliefords		Moran		Shape	
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Skewness	Kurtosis
Before	0.87962	0.04683	0.17201	0.2717	0.54677	0.008367	-0.0925122	-1.483111
After	0.94844	0.7155	0.14723	0.9177	0.37285	0.2193	0.1703448	-1.292267

Table 1.6 Normal-Exponential Tests and Skewness-Kurtosis.

We can also see in Table 1.6 the calculation of skewness and kurtosis; first, we can see that the before window has a negative skewness (-0.0925122), and then it is positive in the after window (0.1703448). In the case of the kurtosis, both windows present negative values: -1.483111 and -1.292267, respectively, representing fewer outliers and less extreme values than the normal distribution (playkurtic).

1.5 Concluding and work in progress

This thesis project aimed to understand how Chile's main economic indicators behaved and, mainly, to provide statistical tools to help in modeling the interest rate, market return, and pension funds. One of the actions to be developed during the study period was to apply for research projects provided by the Financial Markets Commission (CMF) associated with the objectives presented in the thesis project. This was presented, but unfortunately, the CMF research committee did not accept the proposal due to their institutional priorities and available resources. The response letter can be viewed at Appendix A

At the time of presenting this thesis, the preparation of the third article is still pending; although the work done to date is presented in 1.4, we plan to continue working on it until it is submitted to a journal. The main work plan for Article 3 is to stabilize the estimation scheme for the m-parameter estimator. There are two main issues that need to be addressed. Firstly, when m parameter values are very large, the estimates are not good enough. In this case, we need to penalize these values with respect to the value of m. Secondly, the distribution characteristics of m can also affect the estimation accuracy since we are using the Least Squares Estimation method for m. Both these issues can be better explained from the economic point of view of pension funds in Chile.

The distribution of this thesis work is as follows: Chapter 2 contains the first article already published in its original format used by the journal "Stochastic Analysis and Applications." Chapter 3 presents the second article in its original version submitted to the journal "Communications in Statistics - Simulation and Computation." Chapter 4 contains the main conclusions of this work, then the extra references used in the thesis are presented, and finally, the appendix with the principal codes used in the articles and this work.

Chapter 2

Paper 1: Stochastic approach to heterogeneity in short-time announcement effects on the Chilean stockmarket indexes within 2016-2019

Milan Stehlk, Danilo Leal, Jozef Kiselák, Joshua Leers, Luboš Střelec & Felix Fuders (2023), Stochastic approach to heterogeneity in short-time announcement effects on the Chilean stock market indexes within 2016-2019, Stochastic Analysis and Applications, DOI: 10.1080/07362994.2022.2164508. **See original in Appendix A**

Abstract

We aim to examine stock market returns before and after key events in the U.S. Sino trades between 2016 and 2019. The study tracks Cumulative Abnormal Returns (CAR) of the Índice de Precio Selectivo de Acciones (IPSA or S&P/CLX IPSA is a Chilean stock market index for 26 important events throughout this time period. By testing for both directions and significance of market reaction to said events this study aims to clarify if these events and policy announcements were sufficient to influence local equity markets, and in which direction. A simple analysis of CAR showed 16 negative reactions and 10 Positive reactions. An estimated 13 billion USD in market capitalization was lost as a result. Of the 26 events studied 18 were found to produce statistically significant reactions and 8 did not. The IPSA's reaction to the significant events was mixed with 11 negative reactions and 7 positive reactions. We also checked for the normality of the distribution by robust normality tests and expected returns possess significant asymmetry and above-normal kurtosis. As such on aggregate it can be concluded that Chilean capital markets reacted negatively to the U.S. Sino trade war. We model IPSA in the period 2016-2022, where we can observe qualitative differences before and after 2019. To the best knowledge of the authors, the model of IPSA in this paper is the first attempt in this direction.

Keywords: Cumulative Abnormal Returns, Chilean Capital Markets, Stock Market Data, Event Studies, stochastic model of interest rate.

2.1 Introduction

The United States of America (US) and the Peoples's Republic of China (PRC) are the world's two largest economies, with 2018 Gross Domestic Products (GDP) of 20.5 and 13.6 trillion United States dollars (USD) respectively, according to a study done by the World BankThe World Bank (2018). The afore-mentioned values represent 39.7% of the total global GDP.

According to the Office of United States Trade Representative Office of United States Trade Representative (2018), exchange of goods and services between the two countries had a 2018 value of 737.1 billion USD which represents nearly 1 percent of total global GDP. As of 2018, the trade deficit between the two nations sits at 419.2 billion USD in favor of the PRC. Considering the size and the importance of this trade relationship, economic cooperation between the two powers is of vital importance to overall global prosperity, however in recent years relations home come to an impasse. In addition to the the damage being done to US-SINO relations the “trade war” has strong implications for all world economies as the sum of these two nations represents nearly 40% of the world GDP, and as such, it is implicit that any damage done to their economic growth significantly affects world growth.

As the dispute intensifies it is important to consider the economic “collateral damage” done to third parties, who have little or no interest or in or ability to effect outcomes of this dispute yet suffer a large portion of the brunt of this ordeal. To date a multitude of studies have been carried out to assess the damage to both the US and Chinese economies, yet little or no time has been spent assessing the implications and or impacts on small third-party economies.

In the context of a vast and highly interconnected globalized world economy governments should carefully consider the implications of their policy decisions weighing not only the direct impacts but also the the magnitude of the indirect consequences. As such this study will attempt to assess the impact of the trade war to date on Chile is an ideal candidate considering its strong trade ties to both the US and PRC, and its percentage of trade to GDP which sits very close to the world average, (55,7%) according to a study carried out by the OECD (2018) as well as having strong market ties to both of these countries, making it a good proxy for understanding the effects of the trade war on other regions.

The main goal of this paper is to analyze how a third country is indirectly affected by the problems that the two main economies of the world may have. Here we are using the main stock market indicator of the Chilean Stock Exchange as a variable of such analysis. The manuscript is organized as follows: In the next subsections 2.1.1 and 2.1.2 we discuss studies on the effects of economic protectionism and some previous methodologies applied to measuring the impacts of trade wars. In section 2.2 we introduce the data set and methodologies. In section 2.3 we analyze the CAR value events, we test also for normality by utilizing a robust class of tests. In section 4 we introduce a novel model for changes in trade volumes.

2.1.1 Effects of economic protectionism

Protectionism or the act of seeking to positively influence domestic production and economic performance through the use of government, or other regulations to restrict imports has been around for several centuries at a minimum McGee (1993). These policies have their most recent and more informal origins in 1800s French and English mercantilism. Notwithstanding, to this day there is still significant debate as to whether protectionism is a driver of economic welfare loss, and in the case that it is, to what extent. Another area of debate is who ultimately pays the price of tariffs and other measures.

Strange (1985) suggests that the costs of economic protectionism are not as severe as generally agreed upon. The paper suggests that some of the economic turbulence generally attributed to protectionism such as the depressions of the 1930s and 1980 can actually be traced to financial instability and credit difficulties. It is the view of this paper that the idea of protectionism as a driver of major economic instability is essentially a myth.

Eichengreen and Irwin (2009) however takes a much more severe stance on the effect of protectionism citing a rise in protectionist measures as one of the principal motors driving the great depression. Jacks and Novy (2020) examined trade tendencies and protectionism in the between-war period of the 1930s and concluded that protectionist measures resulted in a dramatic drop in overall trade, a drastic shortening of average route length, and the strengthening of politico-economic trade blocks. This work seems to agree that protectionism has dramatic economic consequences, as well as driving the intensification of political tensions.

Feenstra (1992) examined the specific historical effect of sanctions from a U.S. standpoint analyzing their effect on the rest of the world. It was concluded that U.S. sanctions can be very damaging to foreign economies, specifically considering that they tend to be targeted on specific countries and industries, and as such are capable of causing serious consequences. This paper also concludes that free trade associations tend to promote trade within themselves, while also having the consequence of dampening trade between unaffiliated regions. McGee (1993) examines protectionism from 4 distinct viewpoints, and ultimately concludes that these practices are quite beneficial to the specific sector of the economy to which imports are restricted, however, they are a significant driver of overall welfare loss.

As would be expected, non-entry trade measures seem to be more effective than tariffs which allow goods to be imported with a specific tax on the item, making them less competitive in the market. Horstmann and Markusen (1986) studied the effect of inefficient entry on local production costs and found that both average production costs and price charges increased in the industry are protected. There was a general loss of welfare and consumers paid the costs.

Considering the sheer volume of literature supporting the idea that protectionism is either a driver or a major driver of overall welfare and value loss it seems necessary to conclude that this may be the case.

2.1.2 Previous methodologies applied to measuring the impacts of trade wars and other policy shocks on economies and markets

Addressing the specific consequences of the current trade war Li et al. (2018) used a descriptive methodology comparing the preparedness of the U.S. and China to handle a trade war and then examine existing economic data to assess impacts. It was concluded that the U.S. is better prepared to confront this challenge as far less of its economy relies on Chinese trade both in terms of USD value and % of GDP. It also concluded that the U.S. has seen far less economic hardship as a result and it is well positioned to continue and leverage its position for a good outcome.

The preferred method of estimating the economic and welfare loss created by the trade war appears to be via computed general equilibrium models or (CGE). Over the short course of the trade war a considerable amount of literature estimating its effects using CGE models has been created. Giesecke et al. (2019) attempts to apply one such model to estimate impact on U.S., Chinese, Australian, and overall world economies. This work used the Global Trade Analysis Project (GTAP) CGE model with some modifications and found that the joint effect of trade tariffs (U.S. and Chinses) had an overall negative effect on GDP, employment and consumption in both China and the U.S. but surprisingly had an overall positive effect on Australian GDP and consumption while employment figures remained unchanged. It is questionable whether GDP is really a good indicator of welfare. For further references see Max-Neef (1992) or Fuders et al. (2016). The authors suggest a new index for measuring welfare based on the human-scale-development approach Max-Neef (1992) quantifying the subjective perception of the satisfaction of fundamental human needs.

Wang (2019) also uses a CGE model, in this case, a static CGE model to estimate the level of welfare loss caused. His finding was consistent with the majority of studies cited in this literature review, as it concluded that both countries suffered significant welfare losses with China losing significantly more than the U.S. Shagdar and Nakajima (2018) also attempts to estimate welfare loss using a CGE model with data from the GTAP 9.0 database, and had similar findings, that is to say considerable welfare loss to both economies with a more significant decline in China.

Calí (2018) used a novel partial equilibrium model to estimate the future effects of the trade war on both Chinese and east Asian economies in general. His paper focused on identifying products that could potentially be substituted by competing economies in the region. He concluded that considering tariff levels at the time of publishing there would be a 0.3% drop in Chinese GDP. This paper also estimates that the impact on GDP in many potential substitute markets which would look to replace Chinese production could be significant, with Vietnam, The Philippines, and Cambodia leading the way. In total eight countries in the region could experience GDP growth of over 1% as a result of continued tensions.

Shifting this examination to the subject of market impacts Huang et al. (2019) examined the market value of firms in relation to their exposure to global value chains involving the U.S. and Chinese firms. Their work analyzed the immediate impact on stock prices of firms in a reduced time frame, and then compared real-world results to estimated results based on the idea that firms with greater exposure to global value changes would incorporate these policy shocks into their market valuation. The finding of their paper was that real-world results closely followed expected results, and that U.S. firms with elevated exposure to Chinese supply chains were the most severely affected.

Many authors such as Gursida and Indrayono (2019), Zulfikar and Mayvita (2017), H.M. (2015), YUSUF et al. (2012) have applied methodologies of data analytics involving abnormal returns and cumulative abnormal returns in a specific event window on capital markets in order to assess the impact of policy shocks and other types of events to make judgments on the type and magnitude of the market's reaction. See Figures 2.3 and 2.4 for cumulative abnormal returns of specific events. This methodology is particularly useful in assessing such impacts as it can track the outcome of multiple events or announcements and make a cumulative judgment on the overall outcome. This type of work makes use of portfolio theory first developed by Markowitz (1952) which calculates an expected rate of return for a financial instrument based in its systematic and non-systematic rate of risk. Real-world returns are then compared with expected returns within a specific event window in order to assess the impact of events within that event window.

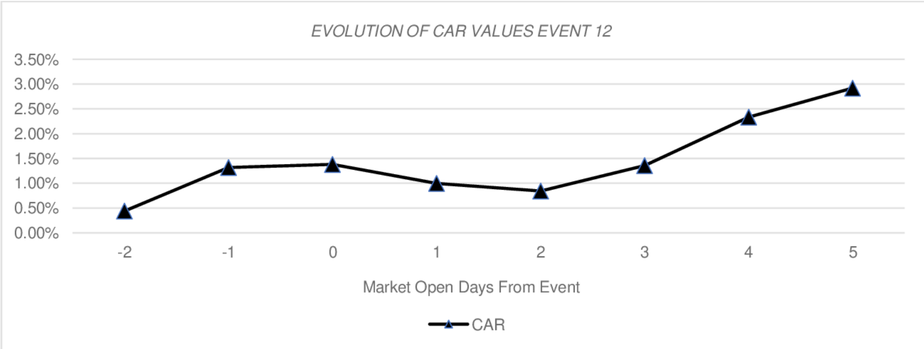


Fig. 2.1 Cumulative abnormal returns for event 12, Source: Santiago Stock Exchange, Authors' Calculations.

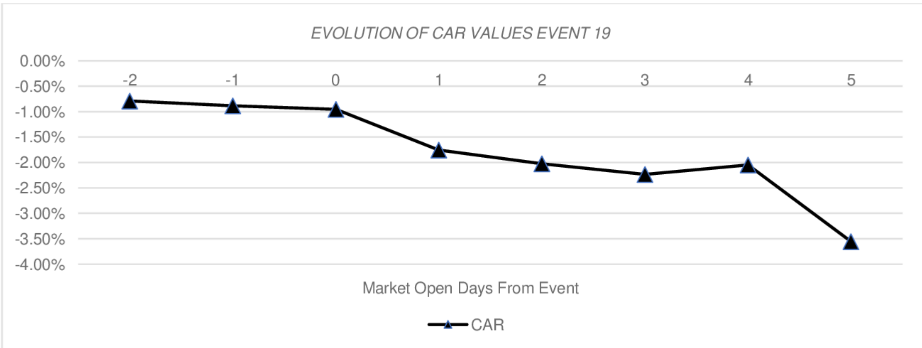


Fig. 2.2 Cumulative abnormal returns for event 19, Source: Santiago Stock Exchange, Authors' Calculations.

2.2 Materials and Methods

2.2.1 Data Description

An initial review of the principal events that composed the trade war revealed 29 significant dates between its origins in June of 2016 when US President announced he would apply tariffs under sections 201 and 301 of the 1974 Trade Act, through October of 2019 when phase one of the trade deal would suspend further tariffs. These events were determined by Timmons (2020). As a matter of judgment, this study has suspended all events with a date after September 10^{th} of 2019 due to significant unrelated market volatility connected to local protests, leaving a total of 26 (3 events were discarded as a result of this decision) events through August 2019.

Having chosen a finite number of events this study compiled data for market performance from the IPSA (Chiles principal stock index) for the aforementioned dates, see Table 2.1 for expected returns. The values obtained were retrieved from the Santiago Stock Exchange (Bolsa de Comercio de Santiago). The data points used were once daily closing values for the event date as well as five market open days after and 2 days before as well as an additional 60 market open days before. This allows for 60 values for the calculation of $E[R]$ (expected return) as well as an 8-day event window. In the event of the market being closed on the day of the event, the next market open day was taken as the event day. A total of 1768 data points were used, although some were repeat values as there was some overlap in event windows. Over the entire data set, the highest daily increase was 6.9%, the lowest daily drop was -5.86% and the average daily return was -0.01%.

Number	**Event Date**	$E[R]$
1	28/6/2016	0.02%
2	31/3/2017	0.27%
3	6/4/2017	0.23%
4	19/7/2017	0.07%
5	14/8/2017	0.08%
6	22/1/2018	0.09%
7	8/3/2018	0.22%
8	2/4/2018	-0.04%
9	3/4/2018	-0.04%
10	4/4/2018	-0.03%
11	15/6/2018	-0.01%
12	10/7/2018	-0.10%
13	1/8/2018	-0.10%
14	7/8/2018	-0.08%
15	24/9/2018	-0.02%
16	1/12/2018	-0.06%
17	1/5/2019	-0.08%
18	3/5/2019	-0.09%
19	5/5/2019	-0.09%
20	16/5/2019	-0.14%
21	18/6/2019	-0.07%
22	29/6/2019	-0.05%
23	1/8/2019	-0.04%
24	5/8/2019	-0.03%
25	13/8/2019	-0.03%

Number	Event Date	$E[R]$
26	23/8/2019	-0.02%

Table 2.1 Expected returns for selected events. Source: Santiago Stock Exchange, Authors' Calculations.

2.2.2 Methodology

The Mean Adjusted Model Method for event studies was used to calculate $E[R]$, essentially a 60-day pre-event average. This decision was made as a result of other methods relying on regression and the calculation of slope, intercept, α, and β, and this is inappropriate when working with a stock index as opposed to a specific stock. $E[R]$ was calculated separately for each event resulting in 26 distinct values. Having compiled the necessary data and calculated expected returns for the period in question this study proceeded to perform statistical analysis. A variant of a methodology commonly used in event studies such as Gursida and Indrayono (2019), Zulfikar and Mayvita (2017), H.M. (2015), YUSUF et al. (2012) was used. This methodology was selected because it is very common in literature for event studies. This methodology determines abnormal returns, cumulative abnormal returns, and average cumulative abnormal returns in order to conclude the magnitude and nature of the effect of an event on market values (negative or positive). This analysis determined the average cumulative abnormal return for each event using the following methodology:

$$\mathrm{AR}_{it} = R_{it} - E[R_{it}],$$

where AR_{it} = abnormal return i on day t of the event window;

$$\mathrm{CAR}_t = \sum_{i=1}^{n} \mathrm{AR}_{it},$$

where CAR_t = cumulative abnormal return for the event;

$$\mathrm{CAAR}_t = \frac{\sum_{i=1}^{n} \mathrm{AR}_{it}}{n},$$

where CAAR_t = cumulative average abnormal return for the event.

CAR values below zero indicate that an event has caused a negative impact, whereas a value over zero would indicate a positive impact. The larger this value the more significant the impact of the event. Values closer to zero indicate that independent of any daily volatility in AR, over the course of the event the impact was neutral.

Additionally, an estimation of monetary variation was made by assessing the impact of CAR for each event on the overall market capitalization of the IPSA at that time. As no historic information of market capitalization was available, but IPSA values were readily available an effort was made to discern the exact relationship between IPSA values and the overall market capitalization of the index. Both IPSA and market capitalization values were registered for several days and linear regression was used to determine the relationship between the two. As the two had a strong correlation ($R^2 = 0.9244$) these values were then used to determine an estimated value for each event ($Y = 27465X - 764032$ with Y being estimated market cap and X being the current IPSA value) which could then be employed in conjuncture with each event CAR value to determine the overall market capital gain or loss as a result.

For the purposes of this study, a comparison of CAAR before and after the event window was used to determine the market reaction. A paired t-test of two samples for means was conducted to test whether there were statistically significant differences in CAR before and after trade war events, and if so what type of reaction had been elicited (positive or negative). t-tests are generally used to determine the chance data is behaving randomly within normal behavior or if a specific event has altered its behavior. Generally, a P value below 0.05 signifies a statistically significant event. The t-test considered an α of 0.05.

The analysis considered a period of 2 market open days before and 5 market open days after each event in addition to the event day itself, as evidence exists that considering longer periods of time tends to create bias in results as shown in Cowan and Sergeant (2001). We calculated the cumulative average abnormal return for a statistically representative population of events within the overall framework of the trade war, and outlined some macro impact of the trade war on Chilean equity markets.

2.3 Results and Analysis

From this point forward each event will be referred to as a number between 1 and 26 according to their date of occurrence with the most recent events having the highest values. For a full table description of events selected see annexed documents. The expected return for each of the events being studied were calculated and the results were as follows:

Having calculated $E[R]$, AR and CAR were now calculated. A sample of 3 events was graphed in order to illustrate the effect of the event on CAR over the course of the time frame studied.

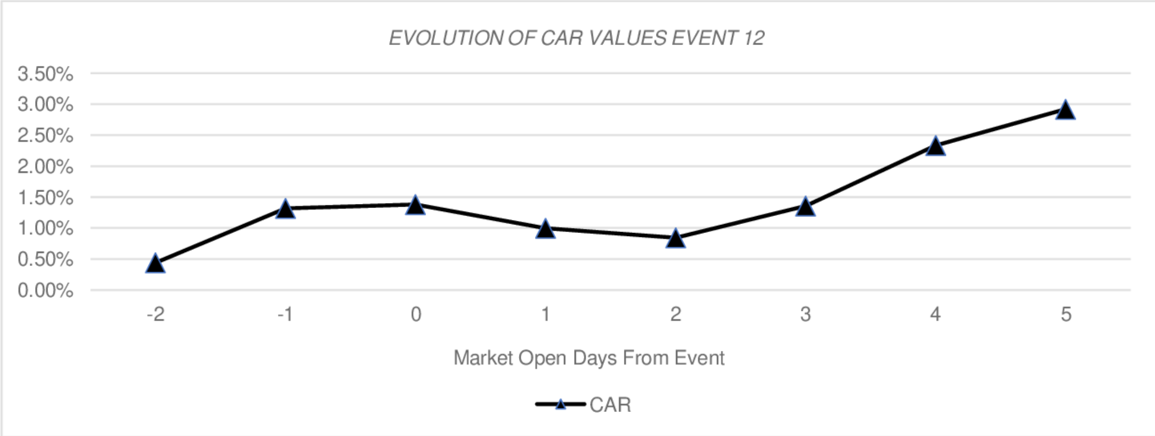


Fig. 2.3 Cumulative abnormal returns for event 12. Source: Santiago Stock Exchange, Authors' Calculations

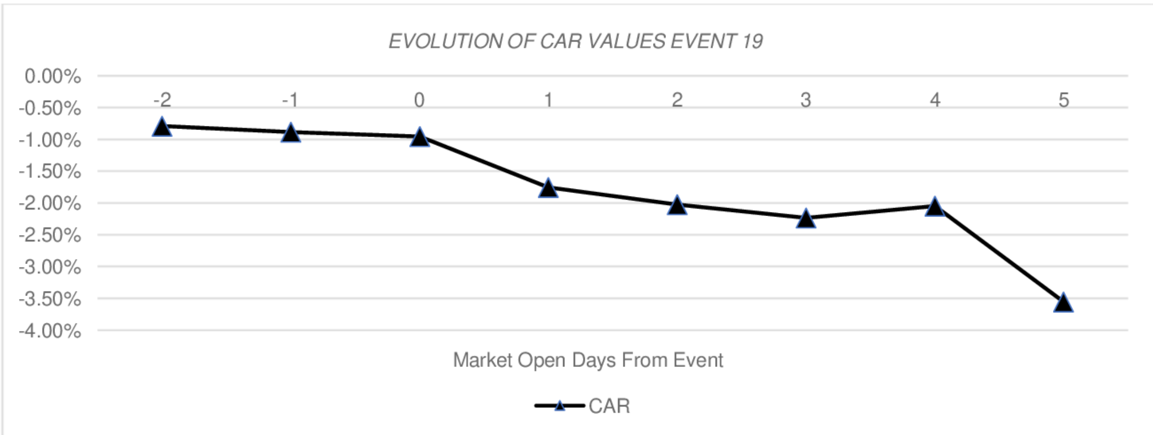


Fig. 2.4 Cumulative abnormal returns for event 19. Source: Santiago Stock Exchange, Authors' Calculations

In the case of event 1 CAR drops strongly in the days before the event, and then makes a strong comeback before finishing the period with a slightly positive CAR at 0.44%. This would indicate that event 1 of the trade war provoked a slightly positive reaction from the IPSA index.

In the case of Event 12, the event begins with a strongly positive reaction in CAR, which dips in the days directly after the announcement but then returns in the last few days, finishing with a CAR of 2.92%. This would indicate that event 1 of the trade war provoked a strongly positive reaction from the IPSA index.

In the case of event 19, the period began with a strong downturn which intensified over the course of the timeframe culminating in a CAR value of -3.56%. This would indicate that event 19 of the trade war provoked a strongly negative reaction from the IPSA index. CAR values for individual periods are given in Table 2.2.

Event	CAR at Event Window End
1	0.44%
2	-1.71%
3	-0.49%
4	0.28%
5	-0.08%
6	0.89%
7	-0.87%
8	3.09%
9	2.89%
10	1.50%
11	-2.97%
12	2.92%
13	-0.16%
14	-1.51%
15	-0.88%
16	-1.09%
17	-2.19%
18	-1.57%
19	-3.56%
20	-0.67%
21	1.12%
22	0.12%
23	-1.43%
24	-2.61%
25	-2.34%
26	0.82%

Table 2.2 Final CAR values for each event period. Source: Santiago Stock Exchange, Authors' Calculations

A simple analysis of CAR value results for the last day of each event without considering statistical significance or pre-event tendency reveals 16 negative market reactions as well as 10 positive reactions. This would indicate that although some events were processed positively by the market, but the overall reaction was negative, with an aggregate value of -10,04%.

Applying the percentual variation in CAR for each event to the total market capitalization at that time permitted the estimation of total IPSA market capitalization loss or gain as a result of the trade war. Variations in market Capitalization are plotted in Table 2.3. Event impact on CAAR pre- and post-event are in Table 2.4.

Event	IPSA	Market Cap. Thousands USD	Event CAR	Variation Thousands USD
1	3936	$107.339.032	0.4%	$474.624
2	4783	$130.612.598	-1.7%	($2.231.554)
3	4898	$133.749.651	-0.5%	($652.007)
4	5037	$137.585.962	0.3%	$388.299
5	5064	$138.311.312	-0.1%	($110.806)
6	5828	$159.299.516	0.9%	$1.418.488
7	5576	$152.390.970	-0.9%	($1.330.471)
8	5503	$150.364.877	3.1%	$4.642.805
9	5534	$151.238.264	2.9%	$4.368.971
10	5543	$151.468.695	1.5%	$2.279.372
11	5470	$149.478.307	-3.0%	($4.440.479)
12	5323	$145.440.128	2.9%	$4.245.695
13	5398	$147.502.200	-0.2%	($231.364)
14	5328	$145.579.375	-1.5%	($2.192.000)
15	5386	$147.152.845	-0.9%	($1.295.837)
16	5152	$140.726.585	-1.1%	($1.534.877)
17	5142	$140.449.463	-2.2%	($3.079.299)
18	5132	$140.195.137	-1.6%	($2.194.649)
19	5124	$139.972.396	-3.6%	($4.976.034)
20	4978	$135.949.872	-0.7%	($915.888)
21	5041	$137.675.223	1.1%	$1.540.743
22	5063	$138.288.517	0.1%	$160.430
23	4941	$134.935.315	-1.4%	($1.923.420)
24	4780	$130.529.379	-2.6%	($3.412.119)
25	4846	$132.320.372	-2.3%	($3.091.626)
26	4649	$126.910.316	0.8%	$1.046.308

Table 2.3 Variation in market Capitalization. Source: Santiago Stock Exchange, Authors' Calculations.

The Cumulative effect on market capitalization was determined to be a net negative equivalent to a loss of approximately 13 billion USD in market valuation.

Having calculated both AR and CAR for each of the events a paired t-test of two samples for means was conducted in order to determine the impact of the event (negative or positive) as well as whether the event provoked a statistically significant response in markets.

Event Identifier	CAAR PRE	CAAR EVENT	DIFF.	Reaction
1	0.22%	-0.01%	-0.24%	Negative
2	1.11%	-0.01%	-1.12%	Negative
3	-0.04%	-0.14%	-0.10%	Negative
4	1.19%	0.35%	-0.84%	Negative
5	0.04%	-0.31%	-0.35%	Negative
6	1.12%	0.41%	-0.71%	Negative
7	-2.44%	0.12%	2.56%	Positive
8	-0.97%	1.48%	2.45%	Positive
9	-1.42%	2.06%	3.48%	Positive
10	-1.22%	0.76%	1.98%	Positive
11	1.14%	-1.67%	-2.81%	Negative
12	-1.44%	1.45%	2.89%	Positive
13	1.59%	0.59%	-1.00%	Negative
14	0.50%	-1.06%	-1.56%	Negative
15	1.37%	0.23%	-1.13%	Negative
16	-0.95%	0.38%	1.33%	Positive
17	0.26%	-1.07%	-1.33%	Negative
18	-0.45%	-0.88%	-0.44%	Negative
19	-0.96%	-1.78%	-0.82%	Negative
20	-1.55%	0.07%	1.62%	Positive
21	0.24%	0.07%	-0.17%	Negative
22	0.30%	-0.24%	-0.54%	Negative
23	-0.80%	-2.15%	-1.35%	Negative
24	-1.02%	-2.19%	-1.17%	Negative
25	-2.20%	-1.83%	0.37%	Positive
26	0.53%	-1.77%	-2.30%	Negative

Table 2.4 Event impact on CAAR pre- and post-event.Source: Santiago Stock Exchange, Authors' Calculations.

Event Identifier	t-stat	$P(T \leq t)$ two-tail	t-Critical two-tail	Event Type
1	0.90	0.399	2.36	Non-Sig.
2	3.50	0.010	2.36	Significant
3	0.24	0.818	2.36	Non-Sig.
4	1.71	0.130	2.36	Non-Sig.
5	1.76	0.122	2.36	Non-Sig.
6	4.26	0.004	2.36	Significant
7	-3.52	0.010	2.36	Significant
8	-3.85	0.006	2.36	Significant
9	-6.60	0.000	2.36	Significant
10	-4.25	0.004	2.36	Significant
11	6.02	0.001	2.36	Significant
12	-9.56	0.000	2.36	Significant
13	2.76	0.028	2.36	Significant
14	5.09	0.001	2.36	Significant
15	1.19	0.272	2.36	Non-Sig.
16	-5.43	0.001	2.36	Significant
17	9.52	0.000	2.36	Significant
18	3.31	0.013	2.36	Significant
19	2.98	0.021	2.36	Significant
20	-10.39	0.000	2.36	Significant
21	0.61	0.564	2.36	Non-Sig.
22	2.18	0.065	2.36	Non-Sig.
23	3.19	0.015	2.36	Significant
24	3.20	0.015	2.36	Significant
25	-0.84	0.431	2.36	Non-Sig.
26	3.84	0.006	2.36	Significant

Table 2.5 t-test for determination of significance. Source: Santiago Stock Exchange, Authors' Calculations.

Upon examination of the two-tail paired t-test of two samples for means and the difference in CAAR values pre and during the events it was concluded that 18 of 26 events studied provoked statistically significant reactions from the IPSA index, see Table 2.5. Among those results ($n = 18$) considered to be significant 11 provoked a negative change in CAAR when compared with pre-event CAAR, 7 pushed the market into a positive reaction. Considering the results of CAR, CAAR and the t-test it can be concluded that on aggregate the U.S. Sino trade war had a strongly negative effect on the IPSA when compared with periods of normal market performance.

2.3.1 Robust testing for normality

The distribution of stock market returns may not follow a normal distribution, and this can vary based on the period analyzed and the frequency of price sampling used to calculate returns. For e.g. a return distribution that contains returns realized during the financial crisis will be very different than one covering a different period. However, expected returns in our case are far from the normal distribution, mainly due to the presence of outliers – see the first boxplot and histogram in Figure 2.5.

For the purpose of testing for normality of the above-presented data sets, we use selected classical tests for normality as well as selected robust trimming techniques normality test (see Stehlík et al. (2014b)). So, in total, we used four classical tests for normality – Anderson-Darling (AD) test, Jarque-Bera (JB) test, Lilliefors (LT) test, and Shapiro-Wilk (SW) test. For the purpose of comparison, we also used four robust tests for normality – robust Jarque-Bera (RJB) test, medcouple (MCLR) test, and two variants of the RT class tests introduced by Stehlík et al. (2014b).

The descriptive statistics are presented in Table 2.6 and boxplots and histograms are in Figure 2.5. Only the first dataset, expected return E[R] is characterized by significant asymmetry, above-normal kurtosis, and the presence of outliers. Therefore, all tests reject the null hypothesis of normality of distribution, at a 5% significance level – see Table 2.7 for p-values of tests for normality for the analyzed datasets. However, at the 1% significance level, some tests do not reject the null hypothesis – specifically the Jarque-Bera test, the medcouple test, MMR1, and MMRT2 tests that are more robust than other tests.

	min	median	mean	max	sd	skewness	kurtosis
E[R]	-0.14	-0.04	-0.01	0.27	0.11	1.38	4.03
CAR at Event Window End	-3.56	-0.58	-0.39	3.09	1.78	0.34	2.50
Event CAR	-3.60	-0.60	-0.40	3.10	1.78	0.35	2.52
CAAR pre-event	-2.44	0.00	-0.23	1.59	1.14	-0.16	1.99
CAAR post-event	-2.19	-0.01	-0.27	2.06	1.15	-0.05	2.30

Table 2.6 Descriptive statistics for the analyzed data sets.

test	E[R]		1		2		3		4	
	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
AD	1.768	0.000	0.223	0.814	0.226	0.804	0.429	0.293	0.539	0.154
JB	9.430	0.013	0.783	0.572	0.770	0.579	1.207	0.363	0.539	0.711
LT	0.248	0.000	0.082	0.922	0.083	0.916	0.122	0.395	0.142	0.191
RJB	32.977	0.005	0.632	0.618	0.629	0.620	1.030	0.399	0.139	0.923
SW	0.825	0.000	0.970	0.618	0.970	0.627	0.955	0.311	0.951	0.247
MC_{LR}	6.801	0.047	0.492	0.910	0.484	0.912	3.674	0.234	3.995	0.200
MMRT1	8.524	0.011	0.931	0.517	0.952	0.508	1.955	0.218	1.378	0.352
MMRT2	6.055	0.015	0.977	0.507	0.998	0.498	2.216	0.180	1.665	0.283

Table 2.7 Results of testing for normality for the analyzed data sets, 1=CAR at Ev. Wdw. End, 2=Ev. CAR, 3=CAAR pre-event, 4=CAAR post-event.

2.4 Modeling of changes in trade volumes

In this section, using the delta method constructed confidence interval, we will show significant statistical differences between trading volumes measured by IPSA before and after the changepoint. Consider a probability space $(\Omega, \mathscr{F}, P)$ and a measurable space (S, Σ), on which a stochastic process lives, i.e. a collection of S-valued random variables, which can be written as $\{r(t,\omega) : t \in T\}$ (to reflect that it is actually a function of two variables). We usually use shorten notation r_t.

Assume that IPSA r_t evolves as a real-valued stochastic process. Notice that at each point in time, the expectation $m(t) := E[r_t]$ of the random variable is the mean (we assume here only L^2 processes). Thus, the mean and also variance $w(t) = E[r_t^2] - E[r_t]^2$, in general, is a function of time.

We now define a new statistical quantity, aggregated IPSA for $u, v \in T, u < v$

$$RC(u,v) := \int_u^v m(t)\,\mathrm{d}t \tag{2.1}$$

In addition to classical power function $x^k, x \in \mathbb{R}, k \in \mathbb{N}$, we use the notation $x^{k^*} := |x|^k \operatorname{sgn}(x), x \in \mathbb{R}$ is a signed power function which guarantees that power $k \geq 0$ might be real for any real values of x. Clearly, functions x^m and x^{m^*} are different for negative values, since x^{m^*} is odd, see e.g. case $m = 2$ on Figure 2.6. Suppose that $r_0 = A, r_0' = B \in \mathbb{R}$ and σ is nonnegative. For intern local dynamics (of IPSA), as will be clear later, we use Chan–Karolyi–Longstaff–Sanders model (1992) with a fixed parameter k

$$\mathrm{d}p_t = (\theta - \beta p_t)\,\mathrm{d}t + p_t^k \sigma\,\mathrm{d}W_t \tag{2.2}$$

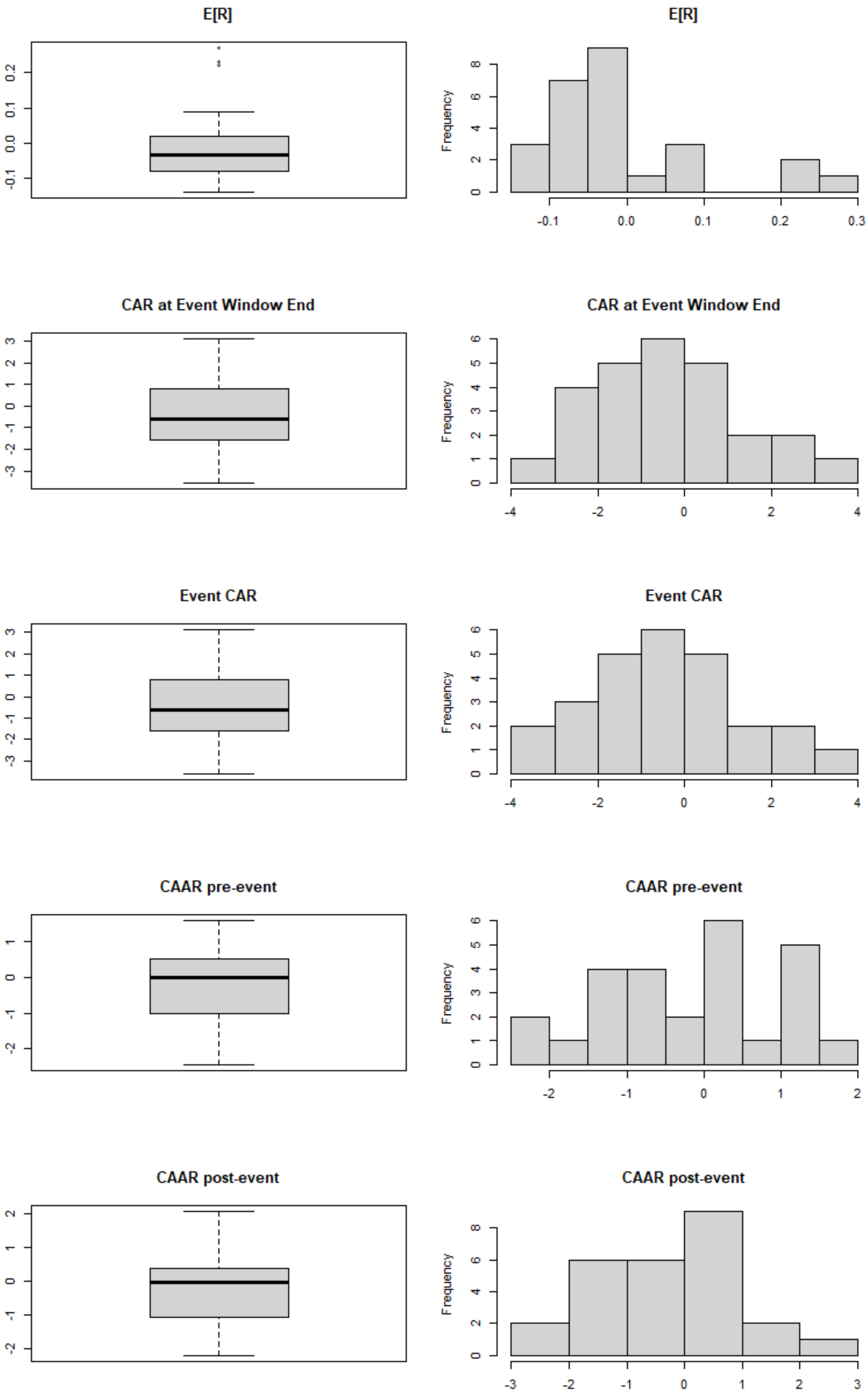
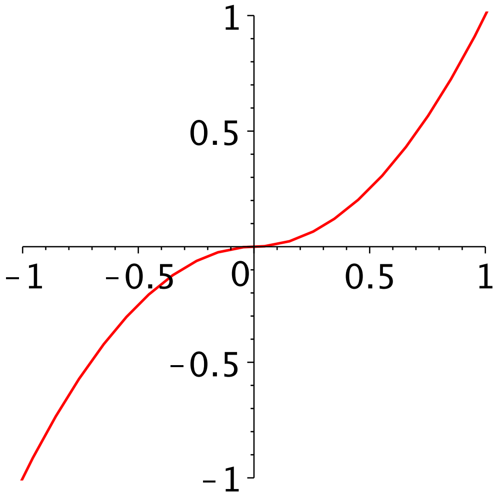
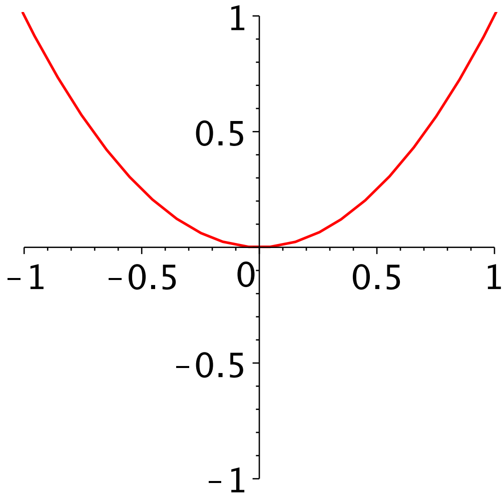


Fig. 2.5 Boxplot and Histograms for the analyzed data sets.

For convergence of interest rate models of this type see Zíková and Stehlíková (2012).



(a) $x^{2^*} = |x|x$



(b) x^2

Fig. 2.6 Graphs of power functions on $[-1, 1]$.

Notice that (2.2) involves several known processes. The Cox–Ingersoll–Ross model supposes $k = 1/2$, the Geometric Brownian motion model supposes $k = 1$ and $k = 0$ implies the famous Ornstein-Uhlenbeck model or the Vasicek model. We focus later here on the last case.

Notice that the stock market index, is an index that measures a stock market, or a subset of the stock market, that helps investors compare current stock price levels with past prices to calculate market performance. Typically stock market dynamics are modeled by a Stochastic Differential Equation. Here we consider that the IPSA is driven by the following two IVPs (two-factor models), where (2.2) is explicitly included for $a = -\beta$ and $b = \theta$. For a better understanding and its good properties see Stehlík et al. (2017) or Kiseľák et al. (2017).

$$\begin{aligned} dr_t &= \left[f\, r_t + c\, p_t^{m^*} + e\right] dt, \\ dp_t &= [a\, p_t + b]\, dt + \sigma\, p_t^k\, dW_t, \\ r_{t_0} &= A,\ r'_{t_0} = B. \end{aligned} \tag{2.3}$$

This model is unique in the way of raising power by p_t. For classical power p_t^m and more general settings see e.g. Stehlík et al. (2017) or Kiseľák et al. (2017). Notice that by changing the assumption of raising power three situations could arise. The solution (with common values of all parameters) could coincide, could coincide only for a specific interval, or could be different on the whole interval of existence, see Stehlík et al. (2017). Notice that the value of m is in the role of stabilization of the process's speed and may influence the value of σ, which is a very interesting fact.

Now, for simplicity, we suppose that $k = 0$, and we are also forced to consider that $c \neq 0$. We obtain specific time-integral of signed powered Ornstein-Uhlenbeck process. The system (2.3) reduces to the nonlinear model, in which r_t can be found explicitly, see Stehlík et al. (2017) or Kisel'ák et al. (2017), e.g. for $t_0 = 0$ we have
$r_t = \mathrm{e}^{ft} \int_0^t \mathrm{e}^{-fs} \left(\left(\sigma \int_0^s \mathrm{e}^{-av} W_v + \frac{b\left(1-\mathrm{e}^{-as}\right)}{a} + \beta_m \right)^{m^*} \mathrm{e}^{mas} c + e \right) s + A\mathrm{e}^{ft}$, where $\beta_m = \left(\frac{B}{c}\right)^{\frac{1}{m}^*}$.

We also assume that $f = 0$ and $m = 1$ are in order to obtain estimable parameters problems. Notice also that the proper estimation of m is quite a difficult open problem. In addition, in the case when $m = 1$, the model (2.3) reduces to the following system

$$\begin{aligned} \mathrm{d}r_t &= [c\,p_t + e]\,\mathrm{d}t, \\ \mathrm{d}p_t &= [a\,p_t + b]\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \\ r_{t_0} &= A,\ r'_{t_0} = B. \end{aligned} \tag{2.4}$$

which is a special case of the linear stochastic equation (for more details see Karatzas and Shreve (1991)):

$$\begin{aligned} d\mathbf{X}_t &= (\mathbf{A}(t)\,\mathbf{X}_t + \mathbf{a}(t))\,\mathrm{d}t + \boldsymbol{\sigma}(t)\,\mathrm{d}\boldsymbol{W}_t,\ \ t_0 \le t < \infty, \\ \mathbf{X}_{t_0} &= \boldsymbol{\xi}, \end{aligned} \tag{2.5}$$

where $d \times d, d \times 1$ and $d \times r$ matrices (in our case $d = 2$ and $r = 2$) $\mathbf{A}(t), \mathbf{a}(t)$ and $\boldsymbol{\sigma}(t)$ are nonrandom, measurable, and locally bounded, one can obtain an explicit solution in the form

$$\mathbf{X}_t = \mathbf{\Phi}(t) \left[\mathbf{\Phi}(t_0)^{-1} \boldsymbol{\xi} + \int_{t_0}^t \mathbf{\Phi}(s)^{-1}\,\mathbf{a}(s)\,\mathrm{d}s + \int_{t_0}^t \mathbf{\Phi}(s)^{-1}\,\boldsymbol{\sigma}(s)\,\mathrm{d}\mathbf{W}_s \right], \tag{2.6}$$

where $\mathbf{\Phi}$ is a fundamental matrix, i.e. the matrix solution of the problem $\mathbf{\Phi}(t)' = \mathbf{A}(t)\mathbf{\Phi}(t)$. Clearly

$$\mathbf{m}(t) := E[\mathbf{X}_t] = \mathbf{\Phi}(t) \left[\mathbf{\Phi}(t_0)^{-1} \boldsymbol{\xi} + \int_{t_0}^t \mathbf{\Phi}(s)^{-1}\,\mathbf{a}(s)\,\mathrm{d}s \right] \tag{2.7}$$

2.4.1 Fitting the model

Here we focus on parameters of the model (2.4). Since it is a special case of (2.5) we have $\boldsymbol{\xi} = (A,B), \boldsymbol{a}(t) = (e,b), \boldsymbol{\sigma}(t) = (0,\sigma)$ and $\mathbf{\Phi}(t) = \begin{pmatrix} 1 & \frac{c}{a}\mathrm{e}^{at} \\ 0 & \mathrm{e}^{at} \end{pmatrix}$, and from (2.6) and the first line in (2.7) we have

$$r_t = m(t) + \frac{c\sigma}{a}\int_{t_0}^{t} (\mathrm{e}^{a(t-s)} - 1)\,\mathrm{d}W_s$$

and

$$m(t) = \frac{c\,(Ba+b)\,\mathrm{e}^{a(t-t_0)} + (e\,(t-t_0)+A)\,a^2 - ((t-t_0)\,b+B)\,ca - bc}{a^2} \tag{2.8}$$

Notice that for $a \to 0$ we have from (2.8) that $m(t) \to (Bc+e)(t-t_0) + A + \frac{bct^2}{2} - t_0bct + \frac{bct_0{}^2}{2}$ and for $c \to 0$ that $m(t) \to e(t-t_0)+A$. Variance can be easily computed by using Ito isometry on $E[r_t^2]$ yielding

$$w(t) = \frac{c^2\sigma^2}{a^2}\int_{t_0}^{t} (\mathrm{e}^{a(t-s)} - 1)^2\,\mathrm{d}s = \frac{c^2\left(\mathrm{e}^{2a(t-t_0)}/2 - 2\,\mathrm{e}^{a(t-t_0)} + 3/2 + a\,(t-t_0)\right)\sigma^2}{a^3} \tag{2.9}$$

with $w(t) \to c^2\sigma^2\left(1/3t^3 - t^2t_0 + t\,t_0{}^2 - 1/3\,t_0{}^3\right)$, if $a \to 0$ and $w(t) \to 0$, if $c \to 0$.

We have fixed the time-change point as 42 due to a 6-month delay due to the COVID period, thus we have two problems with different estimated parameters we have used (2.8) and for given experimental data procedure `NonlinearFit()` from package `Statistics` in software `Maple 2019`):

i) $t_0 = 31, A = r_{31} = 5434.44, B = r_{32} - r_{31} = -164.00 \Longrightarrow \hat{a} = -1.800, \hat{b} = -14.858, \hat{c} = 2.052, \hat{e} = 4.847$.

ii) $t_0 = 43, A = r_{43} = 4972.36, B = r_{44} - r_{43} = -167.99 \Longrightarrow \hat{a} = -0.296, \hat{b} = -124.795, \hat{c} = 0.399, \hat{e} = 17.216$.

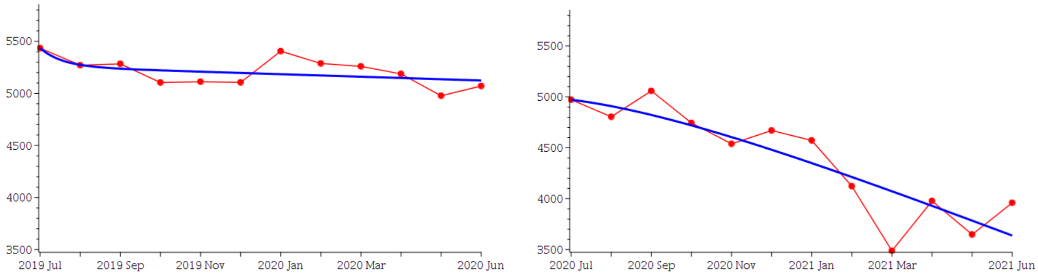
On Figure 2.7 one can see the situation before and after the threshold time-point with values from i) and ii) respectively.

Now, based on the data-driven confidence intervals approach for diffusion processes Yang and Song (2019) and nonparametric delta method Wasserman (2006), we consider $100(1-\alpha)\%$ asymptotic normal confidence interval of the form

$$\left(RC(u,v) - z_{1-\alpha/2}\sqrt{V(u,v)},\ RC(u,v) + z_{1-\alpha/2}\sqrt{V(u,v)}\right), \tag{2.10}$$

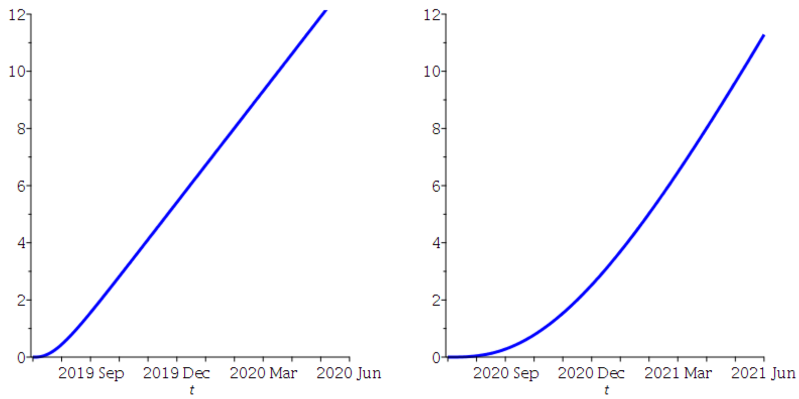
where $V(u,v) = \int_u^v w(t)\,\mathrm{d}t$ is the aggregated variance. We also assume that σ is for both periods in i) and ii) equal to 1. See also Figure 2.8 where variances given by (2.9) with estimated parameters from i) and ii) are plotted. Notice also that in the definition of RC and V can time averaging be used by dividing it by the interval width. In our setup and for $\alpha = 0.05$, we receive two non-overlapping confidence intervals, for choice $z_{1-\alpha/2} = 1.96$.

Namely, these intervals are $(57176.91, 57209.10)$ before, and $(48184.98, 48210.48)$ after the change-point time.



(a) Before (b) After

Fig. 2.7 Fitting of $m(t)$ on two real data periods.



(a) i) (b) ii)

Fig. 2.8 Graphs of $w(t)$ for two different periods.

Notice that estimation of parameters a, b and thus also of σ can be obtained from data $p_{t_i} = \frac{r_{t_{i+1}} - r_{t_i}}{\delta}$ (in the sense of discretization with suitable positive δ) by MLE or OLS for OU process, see e.g. Tang and Chen (2009).

2.5 Discussion and Conclusions

As resumed in paragraph 1.1, economic theory lets little doubt that protectionism and trade barriers can negatively affect *allocative efficiency* and hence provoke overall welfare losses. This paper contributes to the discussion in economic theory giving evidence that welfare losses do not only occur in the economies that are the targets of the protectionist measure but even in third economies, which are not directly part of the ‘trade war’, here Chile.

However, the impact of the US-Chinese trade war on the stock markets in Chile (and probably elsewhere) is not necessarily at all times the same. The world financial system sliders into crisis at regular intervals. Evidence exists that the long economic waves, the so-called Kondratieff waves, are, in fact, cycles of the financial system (Fuders et al. (2013a), Fuders and Max-Neef (2014b)). Symptoms that indicate that we are close to the next mega-crisis, comparable to the Great Depression, are money supply and debt having reached unprecedented levels in all industrialized economies and, associated herewith, the increasing number of credit defaults and speculative bubbles on stock and real estate markets (as well as other perceived 'save havens' such as gold or cryptocurrencies).

The reason why the financial system falls into deep crisis at regular intervals is unfortunately not well understood in economics. A cause could be the unnatural design of our money, see Fuders (2021b) and Fuders et al. (2016). The closer we get to the inevitable collapse Fuders and Max-Neef (2014a) and the bigger the price bubbles on stocks and other investment markets already are, the higher the probability that markets can be impacted by negative trade signals such as trade wars. Hence, it is not necessarily protectionism that causes the statistically significant market reactions, but these could be merely triggered by such events. On the other hand, the closer the world financial system gets to collapse, the more nervous become politicians and it is more likely that protectionist measures will be applied. This could explain the positive correlation between protectionism and the Great Depression authors [3] and [10] mentioned.

It would be interesting to conduct the same study after the next crisis and the reset of the financial system, i.e., when money supply is still low, and we will consequently see less volatility and only a little speculative bubbles on investment markets. Likely, a trade war would then have less impact on the expected returns of stock markets.

This study attempts to determine the impact of policy announcements in the U.S. Sino trade war between 2016 and 2019 on Chilean equity markets. The Mean Adjusted Model Method for event studies was used to determine expected returns and a CAR and CAAR based methodology comparing values within and outside of event windows was used to determine market reactions. This analysis showed a strong negative reaction to the trade war in overall terms. A two-tail paired t-test of two samples for means was employed to discern if the market reaction was statistically significant, and in most cases, it was (18 of 26 events). The majority of those events considered significant (11 of 18 significant events) produced negative reactions on the IPSA. The difference between periods is also confirmed by introduced cumulated measures of trade volumes.

We found that the Chilean stock exchange market reacts to the economic war between the US and China. We can of course suggest that this influence on expected returns is because of the trade ties, however, this does not necessarily need to be the case. It could be that investors at the stock exchange are nervous and so any event in the trade war provokes investors to sell stocks in Chile (and elsewhere). The influence on the Chilean market is not necessarily causal to the trade ties. The market, as measured by the IPSA, has been efficient in processing new information in a timely manner. As such local capital markets performed well pricing in the expectations of new volatility in world trade and adjusting equity valuations accordingly.

One of the limitations of this study methodology is that it gauges market valuations based on expectations of economic results, but not the economic results in and of themselves. A possible opportunity to follow up on this study would be employing a CGE or other economic assessment model to measure GDP or other variations. Additionally, studies could be performed to assess variations in firm earnings in relation with the trade war, and additionally compare market reaction with real world earnings.

Here we used 7 days window, which allows us mainly to concentrate on the immediate reaction of the stock market to the announcement of some information. We do not study possible overreactions of the market and delays. We address partially capital issues and the relationship between the current and capital account of the balance of payments is not addressed. Instead of this, we show that domestic (Chilean) capital markets were influenced in a certain direction.

Chapter 3

Paper 2: On testing the changes in trends of IPSA and rates.

Danilo Leal, Luboš Střelec, Felix Fuders & Milan Stehlk, On testing the changes in trends of IPSA and rates . Under review, Submission ID 239128991, Journal Communications in Statistics - Simulation and Computation. **See original in Appendix B**

Abstract

Calibration of interest rate models benefits from grouping data to homogenous classes. Such an approach is typical in many financial time series. Preliminaries have been developed for Cox-Ingersoll-Ross Model models but this issue remains an open problem for many more realistic interest rate models. Here we develop such a strategy for general class interest rate and classes are based on p-value thresholds for testing for normality and gamma distributions. We use as the benchmark financial series of IPSA and its log-returns. We also study the relationship between interest rate and the market returns represented by the IPSA indicator, with positive correlation in some lags which reveals some interesting facts in the contrary to the conventional theory.

Keywords IPSA, interest rate, likelihood ratio test, testing for normality, parameter dependence.

3.1 Introduction

Orlando et al. (2019) provided preliminaries on calibration of Cox-Ingersoll-Ross (CIR) model-based rate by partitions, where individual groups shall follow normal or gamma distribution. This model was introduced by Cox et al. (1985) and is known as a one-factor time-homogeneous model, an improvement of the Vašíček model. The CIR model describes interest rate as a diffusion process $r = (r(t))_{t\geq 0}$ which is the unique solution of

$$dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t), \quad r(0) = r_0 > 0,$$

here k, θ and σ are positive constant parameters. Due to mean reversion, as time becomes large, the distribution of future interest rate will approach a gamma distribution. The work in Stehlík et al. (2015), appendix 1, illustrate much more advanced models of interest rate, considering a problem of testing whether a sample of observations from an interest rate model can be linked to the inter-arrival times given by a counting process. e.g. the counting process coming from Poisson distributions with different parameters. Such a problem has been studied by several authors, see e.g. Brown et al. (2002). Especially, interest is put to

an over-dispersed and under-dispersed data. From the point of view of parsimony, we could think about intensity to be a simple function of time (i.e. piecewise constant intensity function with a finitely many jumps of the selected counting process), $\lambda(t) = \sum_{i=0}^{n} \lambda_i \xi_{[t_i,t_{i+1})}(t)$, here λ_i is a constant value of intensity in the period $[t_i,t_{i+1})$ and $\xi_I(t)$ is a characteristic function of interval I, i.e. for all $t \in I : \xi_I(t) = 1$ and 0 otherwise. To guarantee validity of such an approach in Stehlík et al. (2018) we proved the uniform convergence of approximate cumulative distribution functions to the exact distribution functions of exact likelihood ratio test for hypothesis $H_0 : \lambda = \lambda_i$ on each compact set. Thus, it is not necessary to work with arbitrary complex counting processes, if we can calibrate the model of a piecewise constant intensity function given the data. Several special cases can be of interest before the most general model, namely Null Hypothesis may contain of counting process N_i driven by a single Poisson with scale λ. We can opt for testing against two alternatives:
a) general alternative $H_{A1} : X_i \sim Poiss(\lambda_i)$ and scales λ_i are arbitrary.
b) change point alternative $H_{A2} : N_i \sim Poiss(\lambda_i)$ and $\lambda_i = \lambda_1, i \leq k$ and $\lambda_i = \lambda_2, i > k$. Here we consider two cases, namely $\lambda_1 < \lambda_2$ or $\lambda_2 < \lambda_1$. Such concept was applied in Stehlik (2003) to receive the decomposition of I-divergence to two LR statistics. Several interesting properties of exact likelihood ratio tests have been observed in Ik et al. (2011), from which, in particular case of our two alternatives (albeit a bit modified), ELR and ELR2 tests are of interest.

Since data of IPSA and/or interest rates are statistically dependent but dependence can be of high complexity, we replace it by parameter dependence models, which are related to development of Filus et al. (2018) where relation to series of papers on pseudo-distributions is given. The transformation approach for so called *multivariate pseudonormal distribution* is cited in Samuel Kotz, pages 217-218. Samuel Kotz acknowledge invariance of pseudonormal distributions with respect to pseudo affine transformation. This was to the best knowledge of the authors the first application of parameter transformation method. The version of method cited in Samuel Kotz was defined as follows: Let $T_1,...,T_k$ be k independent normal random variables. Then one applies the transformation

$$X_1 = aT_1$$

$$X_2 = \phi_1(T_1)T_2 + \theta_1(T_1)$$

$$X_3 = \phi_2(T_1,T_2)T_3 + \theta_2(T_1,T_2)$$

$$X_k = \phi_{k-1}(T_1,...,T_{k-1})T_k + \theta_{k-1}(T_1,T_2,..,T_{k-1})$$

Here $a \neq 0$ and ϕ_i, θ_i are real continuous parameter functions, assumed to be positive and nondecreasing with respect to each argument. The inverse of this transformation is easily computable and one can get joint density function of $(X_1,...,X_k)^T$ and conditional density of X_i given $X_1,...,X_{i-1}$. Any multivariate normal density function is a special case of the above multivariate pseudonormal density when we set ϕ_i to be constant and θ_i to be linear. Filus and Filus (2000) and Filus and Filus (2001) gave a reliability motivation for pseudonormal distributions. The idea of this modelling can be explained as "parameter dependence". There were gamma-dsitribution related pre-concept to parameter dependence, where the time-to-failure distribution of the system is assumed to be gamma. Both parameters of gamma distribution are continuous functions of the load such that the mean time-to-failure is a decreasing function of the load. Even before, in Filus (1980) the pre-conception of "parameter dependence" is given.

In this paper we will concentrate on creation of homogeneous data subgroups which can be well used for calibration of IPSA or interest rate. Several alternative approaches could be provided for modeling non-homogeneous Poisson Processes, see e.g. Bayesian approach in Kuo et al. (1996). Let $y_1,\ y_2,\ldots,y_N$ be univariate random interest rates distributed according to the Exponential densities

$$f(y_i|\vartheta) = \lambda_i e^{-\lambda_i y_i}, \text{for } y_i > 0, \tag{3.1}$$

The basic idea of relating normal or gamma distributed sub-classes to the original counting process is through the parametrization of the intensity, e.g.

$$\lambda_i = \lambda(\mu_i, \sigma_i^2), \tag{3.2}$$

for Normal (μ_i, σ_i^2) distributed subgroup and suitable function $\lambda : R \times R^+ \to (0,\infty)$, and

$$\lambda_i = \lambda_g(\mu_i, v_i),$$

for each gamma (μ_i, v_i) distributed subgroup and suitable function $\lambda_g : R^+ \times R^+ \to (0,\infty)$. Here we consider mean-parametrization of gamma distribution.

The paper is organized as follows. In the following section we recall tests used in the paper. In section 3 real data illustrations are given. Here we consider IPSA series and its log-returns and empirical dependencies between the parameters are discussed. In section 4 we provide economical and statistical discussion and conclusions. Technicalities are left for the Appendix.

3.2 Tests for subgrouped data

First, we start with the hypothetical assumption that we can transform our financial data into approximately independent sub-groups. The clustering problem or sub-groups of extensive data are studied in different disciplines and from various points. Mainly, we are interested in financial time series, and we can see the work of Maharaj and Inder (1999), Duncan et al. (2001), Baran and Sönmezer (2013), who assess the forecasting of financial time series, making sub-groups of the data with different approaches. In this section we recall both tests for homogeneity of exponential distribution and robust tests (RT) for normality, which allows us to classify subclasses according to goodness of fit to exponential or normal distributions. The following two subsections are dedicated to these tests.

3.2.1 Likelihood ratio based tests for subgrouped data

The general subpopulation model which is the alternative tested in the exact likelihood ratio test for homogeneity (ELR) proposed by Stehlik (2003) assumes that each observation follows an exponential distribution (3.1) with its own parameter. A more specific case of a subpopulation model is inhomogeneity with an unobserved clustering and a given number of clusters k, this is the alternative of the exact likelihood ratio test for k subpopulations ELRk introduced in Stehlík and Ososkov (2003). For a sample of N independent observations $\mathrm{y} = (\mathrm{y}_1, \dots, \mathrm{y}_N)$, where $\mathrm{y}_i \sim Exponential(\theta_i)$ we consider LR homogeneity test against the alternative $f(\mathrm{y}_1, \dots, \mathrm{y}_N) = \prod_{i=1}^{N} \theta_i exp(-\theta_i \mathrm{y}_i)$, for $\theta_i \neq \theta_j$ for $i \neq j$ (subpopulation model), i.e. $H_0 : \theta_1 = \dots = \theta_N$ versus non H_0 The LR statistics has the form

$$-ln\Lambda_N = Nln(\sum_{i=1}^{N} \mathrm{y_i}) - NlnN - \sum_{i=1}^{N} ln\mathrm{y_i} \tag{3.3}$$

The ELR2 test uses the alternative of two subpopulations, which can be specified by the existence of two nonempty index sets M_1, M_2 such that

$$M_1 \cup M_2 = \{1,...,N\}, \quad M_1 \cap M_2 = \emptyset \tag{3.4}$$

$$\forall j \in M_1 : \lambda^j = \lambda^1, \quad \forall j \in M_2 : \lambda^j = \lambda^2, \quad \lambda^1 \neq \lambda^2. \tag{3.5}$$

Both the mixture and the subpopulation model can be used to model unobserved clustering. In Stehlík et al. (2018) we consider a subpopulation of a general sample, in which it is valuable to testify for individual parameter of a given value, namely $H_0 : \lambda = \lambda_0$ versus

$\lambda \neq \lambda_0$. The exact distribution function and exact density has been derived in Stehlik (2003). The Likelihood ratio tests for the case of Type I and Type II censored data have been studied in Balakrishnan and Stehlík (2015). In Stehlík et al. (2018) we provided a discussion on three approximations of the exact density: a) by Fourier transformation, b) by saddle-point approximation, c) by approximation driven by numerical solution of ODE, where we prove also uniform convergence of obtained sequence of approximating densities to the exact density. In the next section we provide simulation study for ELR tests.

Simulating powers based on the ELR tests for recognizing the sub-grouping

The critical values $c_{1-\alpha}$ are obtain by simulation from the standard exponential or the Dirichlet distribution. The values $c_{1-\alpha}$ are determined as the respective order statistic $c_{1-\alpha} = c_{(M(1-\alpha))}$. First, it is necessary to specify the simulation setup:

1. We generate exponential distribution with parameter $\lambda = 1$, i.e. Exp(1) for null hypothesis and computed ELR and ELR2 test statistics for sample sizes $n \in \{10, 20, 50, 100, 200\}$.

2. We found critical constants of ELR and ELR2 test statistics, i.e. 95% quantiles of the distributions of ELR and ELR2 test statistics. These are given in the following table:

	$n = 10$	$n = 20$	$n = 50$	$n = 100$	$n = 200$
ELR	9.7839	17.4090	38.1869	70.8623	134.2841
ELR2	6.8426	11.1925	23.2535	42.2952	79.0955

Table 3.1 Critical constants of the ELR and ELR2 test statistics for $\alpha = 0.05$

3. As alternative we generate 50% of Exp(1) and 50% of Exp(λ_2) with rate λ_2 where $\lambda_2 \in \{1, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0\}$ and we use critical constants from Table 3.1 above. We compute power of recognizing non-homogeneity (e.g. of change-point for non-homogeneous-Poisson).

4. Tables 3.2 and 3.3 provide sizes and powers of ELR and ELR2 tests

		$n = 10$		$n = 50$		$n = 100$	
λ	mean=$1/\lambda$	ELR	ELR2	ELR	ELR2	ELR	ELR2
1.0	1.000	0.051	**0.050**	**0.050**	0.046	0.053	**0.050**
1.2	0.833	**0.051**	**0.051**	**0.051**	0.053	0.052	**0.049**
1.4	0.714	**0.050**	**0.050**	**0.052**	**0.052**	0.053	**0.052**
1.6	0.625	**0.050**	0.051	0.049	**0.050**	0.055	**0.052**
1.8	0.556	**0.051**	**0.051**	**0.049**	0.048	**0.050**	0.049
2.0	0.500	**0.050**	**0.050**	0.051	**0.050**	**0.050**	**0.050**
2.2	0.455	**0.051**	0.052	0.051	**0.050**	0.054	**0.051**
2.4	0.417	**0.051**	**0.051**	0.049	**0.051**	**0.053**	0.054
2.6	0.385	**0.050**	**0.050**	**0.053**	**0.053**	**0.051**	0.049
2.8	0.357	0.049	**0.050**	**0.050**	0.053	**0.047**	0.046
3.0	0.333	**0.052**	**0.052**	0.049	**0.050**	**0.052**	0.056
3.2	0.313	**0.051**	**0.051**	**0.050**	0.048	**0.049**	**0.049**
3.4	0.294	**0.051**	**0.051**	**0.053**	0.054	0.049	**0.050**
3.6	0.278	**0.050**	0.052	0.052	**0.049**	**0.049**	0.043
3.8	0.263	**0.050**	**0.050**	**0.048**	0.047	**0.050**	**0.050**
4.0	0.250	0.049	**0.050**	**0.051**	**0.051**	**0.050**	**0.050**
5.0	0.200	**0.049**	**0.049**	0.052	**0.051**	0.048	**0.050**
6.0	0.167	0.051	**0.050**	**0.051**	0.052	**0.050**	0.046
7.0	0.143	**0.051**	**0.051**	**0.050**	0.051	0.055	**0.051**
8.0	0.125	0.051	**0.050**	**0.050**	0.051	0.053	**0.050**
9.0	0.111	**0.050**	**0.050**	**0.051**	**0.051**	**0.051**	0.047
10.0	0.100	**0.050**	**0.050**	0.052	**0.051**	**0.053**	**0.053**

Table 3.2 Size of the ELR and ELR2 tests

$\lambda_1 = 1$	$n = 10$		$n = 20$		$n = 50$		$n = 100$		$n = 200$	
λ_2	ELR	ELR2	ELR	ELR2	ELR	ELR2	ELR	ELR2	ELR	ELR2
1.0	0.050	**0.051**	**0.050**	0.049	**0.052**	0.047	**0.053**	**0.053**	0.049	**0.050**
1.2	**0.053**	**0.053**	**0.050**	**0.050**	**0.056**	0.055	**0.055**	0.052	**0.057**	**0.057**
1.4	**0.057**	**0.057**	**0.063**	0.060	**0.068**	0.066	**0.073**	0.067	**0.085**	0.081
1.6	**0.064**	0.063	**0.069**	0.067	0.078	**0.079**	**0.094**	0.092	**0.119**	0.114
1.8	**0.070**	0.069	**0.079**	0.078	**0.109**	0.106	**0.128**	0.124	**0.182**	0.172
2.0	**0.079**	0.078	0.092	**0.095**	**0.131**	0.129	**0.179**	0.172	**0.259**	0.247
2.2	**0.089**	0.086	**0.109**	0.105	**0.156**	**0.156**	**0.238**	0.226	**0.355**	0.349
2.4	**0.099**	0.098	0.122	**0.126**	**0.202**	0.201	**0.300**	0.296	**0.464**	0.452
2.6	**0.109**	0.108	**0.142**	**0.142**	**0.244**	0.241	0.370	**0.371**	**0.568**	0.566
2.8	**0.122**	0.121	**0.163**	0.159	**0.280**	0.278	0.436	**0.439**	**0.670**	0.669
3.0	**0.131**	0.128	**0.182**	**0.182**	0.316	**0.328**	0.513	**0.516**	0.752	**0.759**
3.2	**0.144**	0.141	**0.210**	**0.210**	**0.371**	**0.371**	0.581	**0.590**	0.820	**0.829**
3.4	**0.159**	0.158	0.225	**0.231**	0.410	**0.423**	0.648	**0.663**	0.880	**0.888**
3.6	**0.170**	0.169	0.247	**0.259**	0.460	**0.479**	0.705	**0.718**	0.916	**0.927**
3.8	**0.182**	**0.182**	0.274	**0.279**	0.496	**0.516**	0.752	**0.766**	0.947	**0.956**
4.0	**0.197**	0.195	0.288	**0.305**	0.547	**0.573**	0.801	**0.825**	0.967	**0.975**

Table 3.3 Results of power of the ELR and ELR2 tests for the mixture of two exponentials

3.2.2 RT class tests for normality

The general *RT* class is based on robustification of the classical Jarque-Bera test introduced by Bera and Bera (1981). The general *RT* class test statistic was defined by Stehlík et al. (2012) for purpose of robust testing for normality against Pareto tails and has the following general form

$$RT = \frac{k_1(n)}{C_1}\left(\frac{M_{j_1}^{\alpha_1}(r_1, T_{(i_1)}(s_1))}{M_{j_2}^{\alpha_2}(r_2, T_{(i_2)}(s_2))} - K_1\right)^2 + \frac{k_2(n)}{C_2}\left(\frac{M_{j_3}^{\alpha_3}(r_3, T_{(i_3)}(s_3))}{M_{j_4}^{\alpha_4}(r_4, T_{(i_4)}(s_4))} - K_2\right)^2, \qquad (3.6)$$

where M_j are jth theoretical central moment estimators of the random variable defined as $M_j(r, T(F_n, s)) = \frac{1}{n-2r}\sum_{m=r+1}^{n-r}\varphi_j(X_{(m)} - T(F_n, s))$ for $j \in \{0,1,2,3,4\}$, where φ_j is a tractable and continuous function, where $\varphi_0(x) = \sqrt{\pi/2}|x|$ and $\varphi_j(x) = x^j$ for $j \in \{1,2,3,4\}$, $X_{(m)}$ is the order statistic, $T(F_n, s)$ is a location functional applied to the sample $X_1, X_2, \ldots, X_n$, r and s are the trimming constants for moments and location, respectively, K_1 and K_2 are small-sample variants of mean corrections, C_1 and C_2 are asymptotic constants, α_1, α_2, α_3

and α_4 are exponents, and finally, $k_1(n)$ and $k_2(n)$ are functions of sample size n. In the general RT class we used the following four different location estimators:

- mean: $T_{(0)} = \frac{1}{n}\sum_{i=1}^{n} X_i$,
- median: $T_{(1)} = F_n^{-1}(1/2)$,
- trimmed mean: $T_{(2)}(s) = \frac{1}{n-2s}\sum_{i=s+1}^{n-s} X_{(i)}$, where $X_{(i)}$ is the i-th order statistic of the sample and s is the trimming constant for location,
- pseudo-median: $T_{(3)} = \text{median}_{i\leq j}(X_i + X_j)/2$, i.e. the median of the set $\{(X_1 + X_1)/2, (X_1+X_2)/2, (X_1+X_3)/2, \ldots, (X_1+X_n)/2, (X_2+X_2)/2, (X_2+X_3)/2, \ldots, (X_2+X_n)/2, \ldots, (X_{n-1}+X_n)/2, (X_n+X_n)/2\}$.

Some detailed information about the general RT class test statistic as well as some theoretical results on consistency and asymptotic χ^2-distribution of this statistic and some geometric aspects of robust testing for normality can be found in Stehlík et al. (2012), Stehlík et al. (2014a) and Richter et al. (2017).

By clustering based on power values from all analyzed tests of RT class the following representatives with good properties have been obtained, for more see Stehlík et al. (2014a):

- The mean-median $MMRT1$ test with test statistic:

$$MMRT1 = \frac{n}{18}\left(\frac{M_3(0,T_{(1)}(F_n,0))}{M_2^{3/2}(0,T_{(0)}(F_n,0))}\right)^2 + \frac{n}{24}\left(\frac{M_4(0,T_{(0)}(F_n,0))}{M_2^2(0,T_{(0)}(F_n,0))} - 3\right)^2.$$

- The mean-median $MMRT2$ test with test statistic:

$$MMRT2 = \frac{n}{18}\left(\frac{M_3(0,T_{(1)}(F_n,0))}{M_2^{3/2}(0,T_{(1)}(F_n,0))}\right)^2 + \frac{n}{24}\left(\frac{M_4(0,T_{(0)}(F_n,0))}{M_2^2(0,T_{(1)}(F_n,0))} - 3\right)^2.$$

- The mean-median $TTRT1$ test with trimming $s = r = 0.05n$ with test statistic:

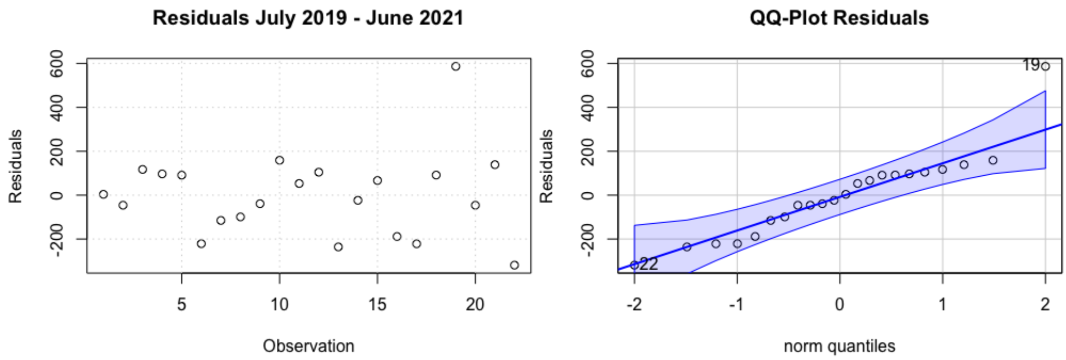
$$TTRT1 = \frac{4n}{5}\left(\frac{M_3(0.05n,T_{(2)}(F_n,0.05n))}{M_2^{3/2}(0,T_{(0)}(F_n,0))}\right)^2 + \frac{27n}{20}\left(\frac{M_4(0.05n,T_{(2)}(F_n,0.05n))}{M_2^2(0,T_{(0)}(F_n,0))} - 0.85\right)^2.$$

- The mean-median $TTRT2$ test with trimming $s = r = 0.05n$ with test statistic:

$$TTRT2 = \frac{16n}{5}\left(\frac{M_3(0.05n,T_{(2)}(F_n,0.05n))}{M_2^{3/2}(0.05n,T_{(2)}(F_n,0.05n))}\right)^2 + \frac{n}{550}\left(\frac{M_4(0,T_{(0)}(F_n,0))}{M_2^2(0.05n,T_{(2)}(F_n,0.05n))} - 7.73\right)^2.$$

3.3 IPSA modeling

The Index of Selective Stock Prices (IPSA for its acronym in Spanish) is the primary stock market performance indicator of the Chilean Stock Exchange that considers the leading traded stocks. Since 2018 it has been called S&P/CLX IPSA after the alliance between S&P Down Jones Indices and the Santiago Stock Exchange. These data were first worked in Stehlík et al. (2023), introducing a model for Ipsa and here we test for the normal distribution of the residuals. The window time is from July 2019 to June 2021.



(a) Residuals (b) QQ-Plot

Fig. 3.1 Residuals.

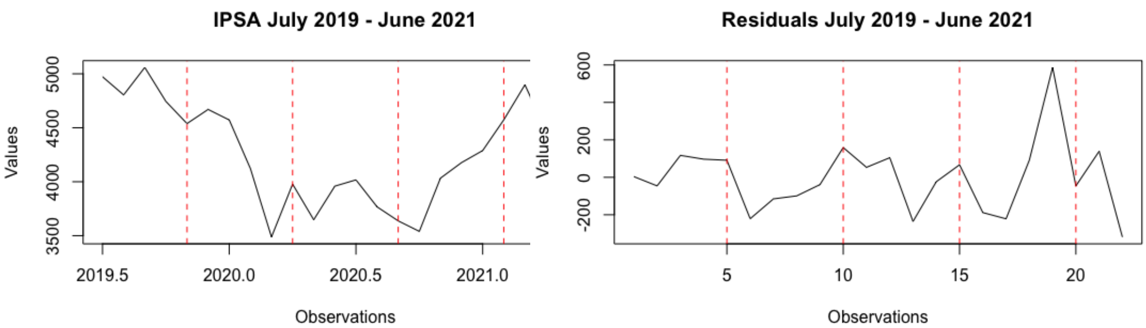
	statistic	p-value
Shapiro-Wilk	0.90258	0.03349
Kolmogorov-Smirnov	0.5	0.00001306
Anderson-Darling	0.58652	0.1133
Lilliefors	0.15433	0.1885

Table 3.4 Residual normally tests.

As we can see in Table 3.4, the results are divided, the last two show normality which is just the opposite result to the first two, and the most different result is the Kolmogorov-Smirnov test. However, this case is vastly studied, and it is not recommended in some instances where we work with a small sample, and when the parameters are estimates from real data, you can see, e.g., Ghasemi and Zahediasl (2012),Steinskog et al. (2007), Marsaglia et al. (2003) and references therein. Continuing with the residual analysis, we apply the methodology presented for Orlando et al. (2019) that divides the dataset into sub-set of data. The algorithm starts with a minimum of observation, in our case, the first five, and then the Lilliefords test is run; if the result is normal, then the algorithm continue with the following five observations. If the Lilliefors test is not normal, the algorithm adds new observations until the test does not rejects normality. This is repeated several times with the entire dataset. We also applied JB, LT, RJB, SW, MMRT1, MMRT2, TTRT1, TTRT2 tests developed in Stehlík et al. (2014a) with satisfactory results of a robust normality.

Data	IPSA		Subset 1		Subset 2		Subset 3		Subset 4	
Test	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
JB	1.144	0.379	0.362	0.783	0.267	0.883	0.579	0.491	1.132	0.094
LT	0.088	0.904	0.154	0.943	0.169	0.873	0.231	0.414	0.217	0.513
RJB	0.868	0.468	0.221	0.844	0.200	0.868	0.457	0.468	1.298	0.153
SW	0.965	0.551	0.975	0.917	0.959	0.813	0.933	0.610	0.859	0.184
MMRT1	1.237	0.390	0.439	0.714	0.156	0.938	0.648	0.523	1.546	0.134
MMRT2	1.278	0.386	0.474	0.696	0.157	0.942	0.683	0.510	1.234	0.178
TTRT1	4.214	0.407	4.211	0.713	10.437	0.389	1.837	0.927	25.643	0.118
TTRT2	1.128	0.497	0.402	0.897	0.593	0.580	0.436	0.822	2.370	0.093

Table 3.5 Results of testing for normality for the analyzed data sets.



(a) Raw IPSA. (b) Residuals.

Fig. 3.2 Subsets Lilliefors Test.

Analyzing a subset of the time series is essential to find clusters of volatility or jumps caused by some announcement of an influential person like in the study in Stehlík et al.

(2023) or decisions by central banks like in the case of the Monetary Policy Rate. In our case (see Figure 3.2), for both the original dataset of IPSA and the residuals from Stehlík et al. (2023), the subset was compounded by five observations showing normality by the Lilliefors test. Table 3.6 presents the p-values for each subset for both data sets.

Subsets	IPSA p-values	Residuals p-values
1	0.9154639	0.1217613
2	0.7210761	0.5064873
3	0.6182405	0.2654715
4	0.6839121	0.4302534

Table 3.6 IPSA and Residuals p-values Lilliefors test.

Following the same idea presented above, we use the same algorithm but, at this time, we apply an exponential test; as a result, specifically the Moran test. The reason to use the Moran test is the fact it is pivotal; see Moran moran Moran (1951), Stehlik (2003), Stehlík (2006). Table 3.7 represents the p-values for raw data for IPSA from June 2019 to July 2021.

Subsets	IPSA p-values
1	0.984
2	0.985
3	0.986
4	0.988
5	0.9845
6	0.987

Table 3.7 IPSA p-values Moran test.

Considering the Moran‘s test, the subsets were compounded by four observations, which we see in the following figure.

3.3.1 Monthly Return Analysis of IPSA

A traditional calculation for working with stock prices or indexes is the logarithm of returns. We can compare different assets based on their profitability and not their price. Below we present the IPSA's return in the analysis period and a histogram that reflects the behavior of its returns. We checked several classical and robust normality tests (JB, LT, RJB, SW,

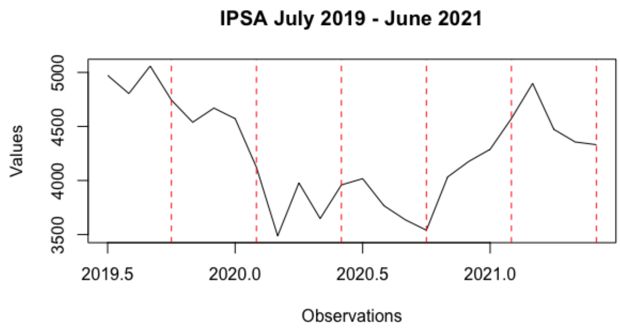
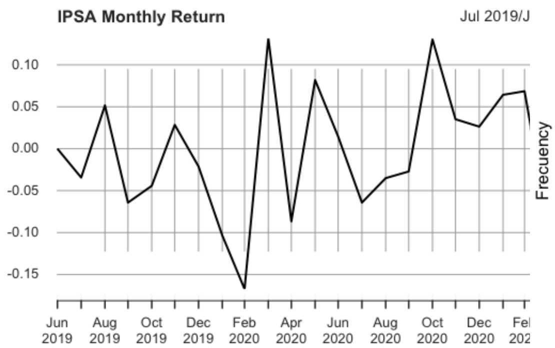
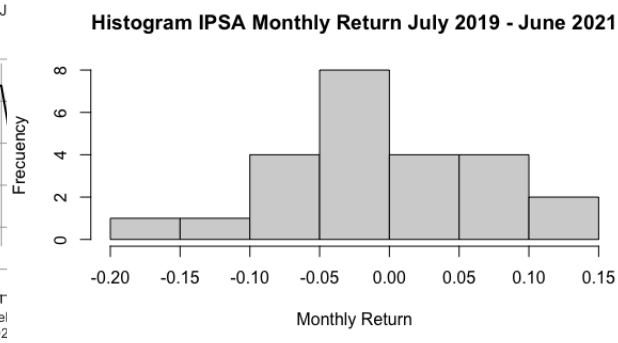


Fig. 3.3 Subsets Moran test.

MMRT1, MMRT2, TTRT1 and TTRT2) for all monthly returns, and also for subsets 1-4 (each having $n = 5$ observations) and null hypothesis of normality was not rejected (see Table 3.8).



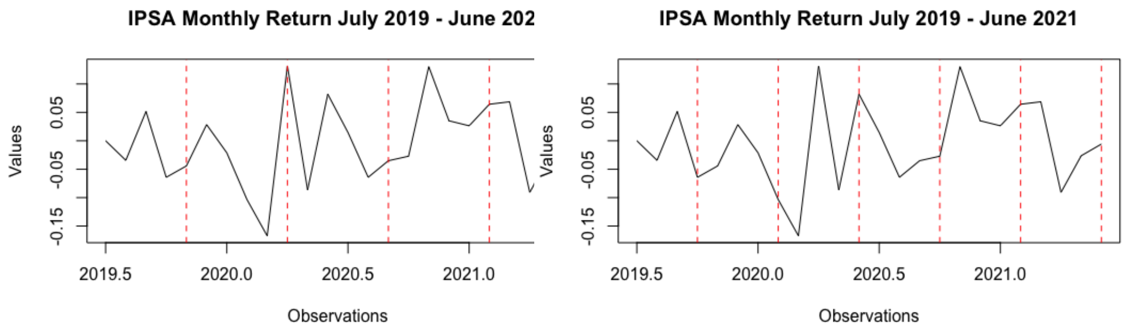
(a) Monthly Return.



(b) Histogram of the Returns.

Fig. 3.4 Monthly Returns of IPSA.

Following the analysis above, we apply the same algorithm with Lilliefors and Moran tests to detect subsets, but this time with the monthly return of IPSA to understand the volatility clusters from an economic point of view. The results of subsets are the same as for raw IPSA.



(a) Monthly Return Lilliefors Test. (b) Monthly Return Moran Test.

Fig. 3.5 Monthly Returns Subsets.

Data	Monthly returns		Subset 1		Subset 2		Subset 3		Subset 4	
Test	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
JB	0.087	0.959	0.537	0.555	0.527	0.568	0.749	0.272	0.449	0.676
LT	0.083	0.938	0.203	0.620	0.196	0.670	0.199	0.653	0.166	0.891
RJB	0.069	0.962	0.412	0.555	0.364	0.641	0.689	0.243	0.344	0.673
SW	0.985	0.962	0.951	0.754	0.945	0.704	0.917	0.486	0.935	0.627
MMRT1	0.232	0.873	0.660	0.512	0.816	0.391	0.868	0.358	0.383	0.764
MMRT2	0.267	0.856	0.706	0.491	0.892	0.351	0.813	0.407	0.391	0.769
TTRT1	2.681	0.558	2.888	0.838	3.523	0.779	19.138	0.190	8.552	0.456
TTRT2	0.882	0.619	0.502	0.684	0.551	0.622	1.619	0.178	0.796	0.434

Table 3.8 Results of testing for normality for monthly returns.

In Table 3.9 we present the parameters for normal and exponential distributions for all subsets; for the normal case, we used regular mean and standard deviation, and for the exponential (a) case, we transform the data linearly with the form $\hat{y} = x + a$ taken arbitrary selection of $a = 1$. We calculate $\hat{\lambda}$ by MLE for this case in each group, for values see Table 9, column 3.

In the exponential (b) case, we use the two-parameter exponential distribution

$$\frac{1}{\sigma}\exp\left(-\frac{(x-\mu)}{\sigma}\right). \tag{3.7}$$

For this case (3.7) we calculated the parameter lambda as follows: $\hat{\lambda}_i = \frac{n-1}{\bar{x}-x_{imin}}$. This is based on results of Epstein (1956), where complete and sufficient statistics for (μ, σ) of is derived as $(x_{(1)}, V)$, $V = \frac{n}{n-1}(\bar{x} - x_{(1)})$, $\bar{x}$ is arithmetic mean and $x_{(1)} := min(x)$. The reason for using two-exponential model (3.7) is that we do not know lower range of values for each group, and since the minimum varies in each group, we shall estimate it. We can see substantially higher variation of the estimated values $\hat{\lambda}_i$, see Table 9, column 4.

Distribution	Normal		Exponential		Exponential (a)	Exponential (b)
Parameter	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\lambda}$
Subset 1	-0.01824638	0.04547658	-0,01174663	0,049757887	1.011886	57.11886722
Subset 2	-0.02639882	0.1159322	-0.03510419	0.054751262	1.036381	43.87993878
Subset 3	-0.01788538	0.06741795	-0.01012370	0.140335861	1.010227	19.08575026
Subset 4	0.04580174	0.05767669	-0.02797310	0.03253285	1.028778	82.72798851
Subset 5	NA	NA	0.06403655	0.04710354	0.9398173	79,55341365
Subset 6	NA	NA	-0.01359393	0.06568482	1.013781	38,84608733

Table 3.9 Estimated parameters of the Subsets.

When we use model of parameter dependence (3.2) for column 4 (Exponential (a)) in Table 3.9, namely $\lambda_i = \lambda(\mu_i, \sigma_i^2)$ by simple ordinaly least squares regression we obtained

$$\hat{\lambda} = 1.0020160 - 0.0118259\hat{\sigma} - 0.9676346\hat{\mu},$$

with Multiple R-squared: 0.9996 and Adjusted R-squared: 0.9993, and for the column 5 (Exponential (b)) , we have:

$$\hat{\lambda} = 88.89 - 523.45\hat{\sigma} + 228.43\hat{\mu},$$

with Multiple R-squared: 0.8123 and Adjusted R-squared: 0.6872.

Setup for simulation

1. We run $M = 100{,}000$ simulations $X \sim exponential(\lambda = 1)$ for n equals the sample size $n = 24$.

2. Running simulation setups for the ELR, ELR2 a ELR3 tests to receive discretization for later computing of p-values.

3. For IPSA dataset we compute the ELR, ELR2 and ELR3 test statistics.

4. Computing of p-values.

	statistic	p-value
ELR	0.1392	1.000
ELR2	0.1020	1.000
ELR3	0.1239	1.000

Table 3.10 Test statistics and p-values of the ELR, ELR2 and ELR3 tests for IPSA data.

IPSA and Interest Rate

The IPSA represents the return of the market as well as other stock market indicators in different stock exchanges around the world, e.g., Ibex 35 in Spain, S&P 500 in the United States, and Bovespa for Brazil, among others. On the other hand, the interest rate represents the return obtained by a financial institution when lending money, and from the consumer's point of view, it is the cost of being able to use the money. The relationship between interest rates and the market return has been studied in different countries, and the result depends on the country analyzed; you can see Canova (1997), Lee (1992), Øystein Gjerde and Sættem (1999) and references therein. For our case, we realize a cross-correlation analysis between current interest rate and IPSA from 2016 to 2022.

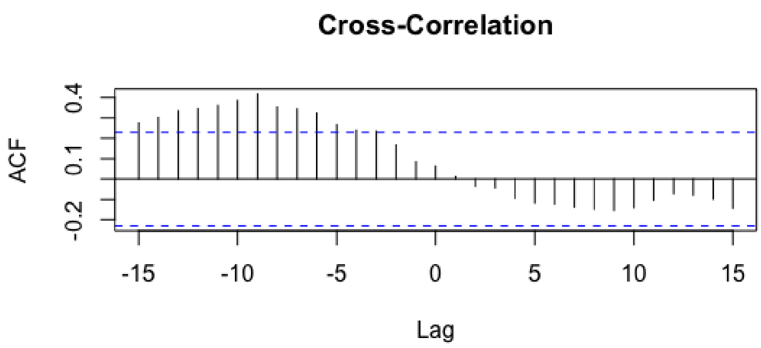


Fig. 3.6 Cross-Correlation (CC).

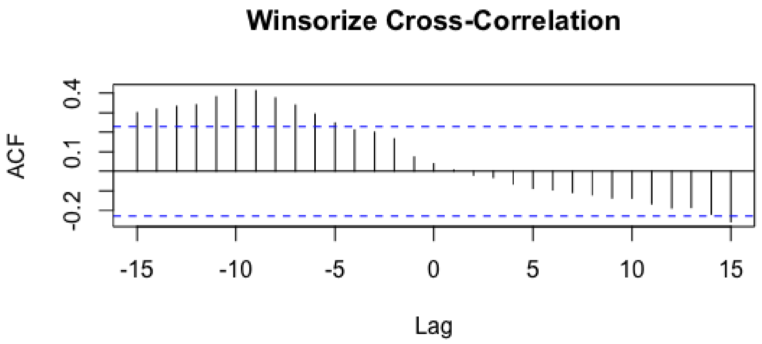


Fig. 3.7 Winzorize Cross-Correlation (WCC).

Figure 3.6 and 3.7 represents the correlation between IPSA and Interest rates at different lags, and Table 3.11 shows the cross-correlations values. We also calculate a robust estimation using R-package datawizard described in Patil et al. (2022). Robustness to outliers is used, using the percentile method with a threshold of 0.2

Lag	CC	WCC	Lag	CC	WCC
0	0.063	0.038	0	0.063	0.038
-1	0.084	0.074	1	0.011	0.006
-2	0.168	0.167	2	-0.036	-0.020
-3	0.235	0.199	3	-0.045	-0.031
-4	0.240	0.210	4	-0.094	-0.066
-5	0.266	0.247	5	-0.119	-0.087
-6	0.324	0.293	6	-0.124	-0.095
-7	0.343	0.338	7	-0.138	-0.108
-8	0.352	0.376	8	-0.148	-0.119
-9	0.417	0.412	9	-0.154	-0.137
-10	0.385	0.417	10	-0.141	-0.138
-11	0.362	0.382	11	-0.104	-0.166
-12	0.344	0.339	12	-0.072	-0.187
-13	0.334	0.332	13	-0.079	-0.184
-14	0.303	0.318	14	-0.098	-0.220
-15	0.275	0.301	15	-0.143	-0.257

Table 3.11 Auto-correlations of series by lag.

We have observed that there is a positive correlation between interest rates and returns in the stock exchange market, at least in several instances where there is a delay between the two. This means that higher interest rates tend to lead to higher returns in the stock market, which is contrary to what is conventionally known.

3.3.2 Formal check of non-zero cross-correlations

Based on high variability we decided to use several correlation measures, namely Pearson, Winsorized, Jackknifed, robust and Spearman. Thus, we can check for all these established correlation measures non-zero cross-correlations. Therefore we used formal test statistics for standard hypothesis of zero cross-correlation

$$H_0 : K_{XY} = 0 \text{ versus } K_{XY} \neq 0, \tag{3.8}$$

The test statistics obtained by Rice and Shum (2019) has the form

$$S_N = N \sum_{i=0}^{p} C(p)^2, \tag{3.9}$$

where $C(p)$ is selected cross-covariance from our portfolio of Pearson, Jackknifed, Spearman, etc and N is the sample size.

3.4 Conclusions and discussion

The obtained results are two-fold. We received simulation and modelling results regarding the new testing model by construction of empirical subgroups and parameter dependence models. Regarding the application of the new testing model on IPSA data the results are quite interesting. We could see that an interest rate and returns at the stock exchange market are positively correlated at least in several lags, i.e., higher interest rates lead to higher returns at the stock market. This is contrary to conventional knowledge. In finance it is taught that the correlation should go the other way around: higher rates would put a pressure on stock market prices. This is because if interest rates are on the rise, investment in stocks become relatively unattractive since investors would then prefer to invest in interest bearing bonds or other debt securities, not in stocks. Figure 3.8 clearly depicts the long-term positive correlation between interest rates and stock market prices. Data provided by Board of Governors; S&P DJI, both retrieved from fred.stlouisfed.org.

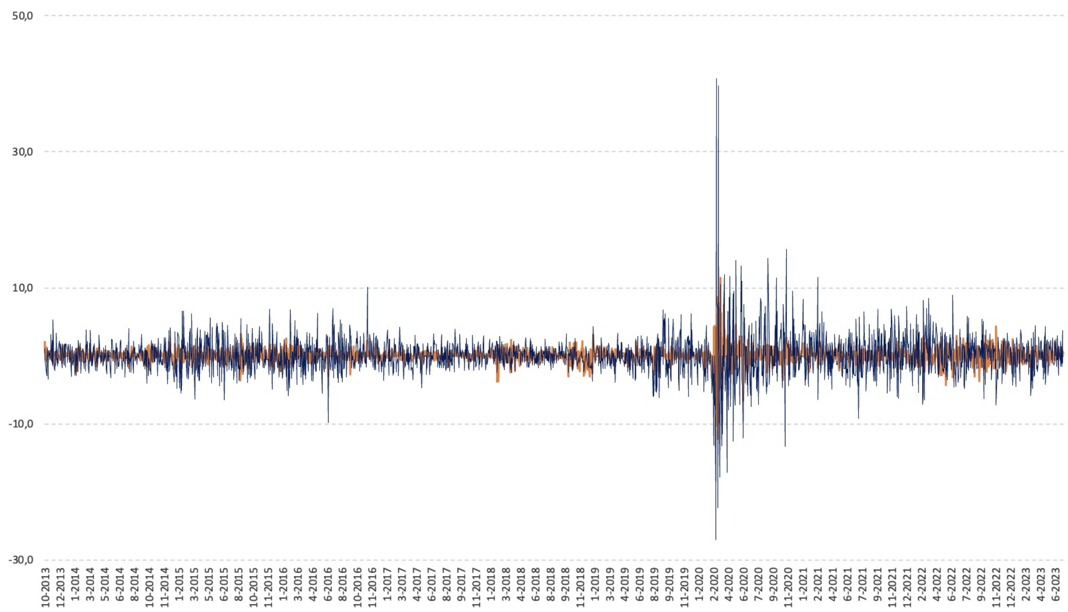


Fig. 3.8 Dow Jones and long-term interest rates (US). Blue: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity (percent change). Red: Dow Jones Composite Average (percent change).

We found a positive correlation between IPSA (stock market) returns and interest rate, and we concluded that this contrary to conventional wisdom in finance. However, we can intuitively understand why this conventional understanding can, at its best, only be true in the very short run. In the medium and long run, higher interest rates lead to deposits grow faster. All deposits taken together integrate the economy's money supply basically consisting of cash plus deposits. There exist different definitions of "money supply". The most commonly one used is the so-called broad money "M3" consisting of cash (bills and coins) and demand deposits (checking accounts) + all time-related deposits, saving deposits, non-institutional money-market funds + large and long-term deposits (2 years), institutional money-market funds, repurchase agreements, along with other larger liquid assets.

The faster money supply grows the more money will ceteris paribus be invested in investment markets such as stock or real estate. Also, to be able to pay the interest on deposits. Credits expansion then again increases the money supply via the so-called "money-multiplier" effect (see, e.g., Mankiw et al. (2024)). This in turn creates a potential for asset price inflation, i.e., an increase in stock market prices. This is why prices investment markets and interest rate in the long run develop synchronously see already Fuders et al. (2013b), Fuders et al. (2014) and Fuders (2021a). We could prove here this heterodox interpretation and show that interest rates and stock market return are indeed positively correlated.

Chapter 4

Conclusions

4.0.1 Stochastic approach to heterogeneity in short-time announcement effects on the Chilean stockmarket indexes within 2016-2019

The research presented in the paper offers significant insights into the impacts of economic protectionism and trade barriers, particularly in the context of the US-China trade war. The study highlights that the negative effects of such measures extend beyond the primary economies involved, affecting third-party economies like Chile. This underlines the interconnected nature of the global financial system, where actions in major economies can have ripple effects across the globe.

The Mean Adjusted Model Method, chosen for its suitability in dealing with stock indices rather than individual stocks, was critical to this analysis. This method facilitated the calculation of expected returns based on a 60-day pre-event average, which is crucial in avoiding the complexities and potential inaccuracies of regression-based approaches. In delving deeper into the market's reaction to these events, the study employed Abnormal Return (AR), Cumulative Abnormal Return (CAR), and Cumulative Average Abnormal Return (CAAR) as its primary analytical tools. These metrics were pivotal in quantifying the magnitude and nature of positive or negative market reactions to each event. A detailed examination of a specific event window revealed a predominance of negative market reactions, indicating an overarching adverse impact of these events on the market.

A crucial aspect of the study was testing the normality of return distributions, a step fundamental in understanding the underlying behavior of stock market returns. The research thoroughly evaluated the distribution characteristics by employing both classical and robust tests for normality, including the Anderson-Darling, Jarque-Bera, Lilliefors, Shapiro–Wilk, robust Jarque-Bera, medcouple, and RT class tests.

In an innovative approach, the study modeled changes in trade volumes using a stochastic differential equation model specifically tailored to the dynamics of the Chilean stock market index (IPSA). This model incorporated various statistical and mathematical techniques, including the Chan–Karolyi–Longstaff–Sanders model and a signed power function for handling negative values. The resulting analysis, buttressed by data-driven confidence intervals, revealed significant statistical differences in trading volumes across these periods. This part of the study provided insights into the market's behavior in response to external shocks and highlighted the utility of advanced statistical models in deciphering complex market dynamics.

4.0.2 On testing the changes in trends of IPSA and rates

The paper delves into the calibration of interest rate models by segmenting data into homogeneous classes, a method widely used in financial time series analysis. This approach was applied to the general class interest rate model, utilizing p-value thresholds to test for normality and gamma distributions. The study primarily focused on the Index of Selective Stock Prices (IPSA), Chile's pivotal stock market indicator. Through this lens, the research tested for the normal distribution of residuals in the IPSA data from July 2019 to June 2021. This methodology underscores the significance of classifying financial data to understand market dynamics better and predict future trends.

A variety of statistical tests were employed in the study. These included tests for subgrouped data to transform financial data into approximately independent sub-groups, the exact likelihood ratio test for homogeneity (ELR) and its variations (ELRk and ELR2) for testing exponential distributions in subpopulations, and robust tests (RT) for normality, specifically a modified version of the Jarque-Bera test. Additionally, the paper utilized various other statistical tests like JB, LT, RJB, SW, MMRT1, MMRT2, TTRT1, and TTRT2 to ensure robust normality in the data. The Lilliefors and Moran tests were also applied to detect volatility clusters within the monthly return of IPSA, providing a deeper understanding of economic fluctuations.

The study's most striking finding was the identification of a positive correlation between interest rates and market returns, as indicated by the IPSA. This result is particularly intriguing as it contradicts conventional financial theory, which posits an inverse relationship between interest rates and stock market performance. According to traditional views, higher interest rates are thought to make stocks less attractive, leading to lower stock market returns. However, the paper's findings suggest that higher interest rates correspond with higher market returns in the context of the IPSA. This discovery challenges established financial doctrines and offers new insights into the complex dynamics of stock markets and interest rates, potentially reshaping our understanding of investment strategies and economic forecasting.

4.0.3 Pension Funds: The Chilean case in times of pandemic

The COVID-19 pandemic brought forth unprecedented challenges for pension funds in Chile. The government's response, allowing withdrawals from pension funds, was necessary to provide immediate financial relief. However, this short-term relief had long-term consequences, such as significant reductions in pension savings and increased fiscal burdens. The situation underscored the importance of having flexible and robust pension systems that can withstand economic shocks.

This section employs a linear econometric model to dissect the factors impacting pension funds, focusing on market return (IPSA), interest rates, and a geometric series (m). Additionally, residual analysis and Cook's distance are particularly noteworthy. This approach helps identify influential data points and anomalies in pension fund yields, a critical aspect for comprehending the volatility and unexpected trends that have emerged during the pandemic.

The gender gap in withdrawals and the depletion of pension savings for many people highlight the need for better financial education and planning. It also raises questions about the pension system's sustainability in the face of such shocks. Moreover, the impact on inflation and monetary policy illustrates the interconnectedness of pension systems with the broader economy. The central bank's response to counteract inflation by increasing interest rates, while necessary, had a cascading effect on other financial sectors, particularly impacting loans.

This situation also revealed the importance of investment strategies within pension funds. The risk profiles of the five types of funds managed by AFPs in Chile demonstrate the need for a balanced approach to risk and return, especially in volatile economic times. However, it also highlights the need for tools to predict market movements and optimize fund allocations for all contributors, regardless of age.

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Appendix A

Paper 1

STOCHASTIC ANALYSIS AND APPLICATIONS
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Stochastic approach to heterogeneity in short-time announcement effects on the Chilean stock market indexes within 2016-2019

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ABSTRACT

We aim to examine stock market returns before and after key events in the U.S. Sino trades between 2016 and 2019. The study tracks Cumulative Abnormal Returns (CAR) of the Índice de Precio Selectivo de Acciones (IPSA or S&P/CLX IPSA is a Chilean stock market index) for 26 important events throughout this time period. By testing for both directions and significance of market reaction to said events this study aims to clarify if these events and policy announcements were sufficient to influence local equity markets, and in which direction. A simple analysis of CAR showed 16 negative reactions and 10 Positive reactions. An estimated 13 billion USD in market capitalization was lost as a result. Of the 26 events studied 18 were found to produce statistically significant reactions and 8 did not. The IPSA's reaction to the significant events was mixed with 11 negative reactions and 7 positive reactions. We also checked for the normality of the distribution by robust normality tests and expected returns possess significant asymmetry and above-normal kurtosis. As such on aggregate it can be concluded that Chilean capital markets reacted negatively to the U.S. Sino trade war. We model IPSA in the period 2016–2022, where we can observe qualitative differences before and after 2019. To the best knowledge of the authors, the model of IPSA in this article is the first attempt in this direction.

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1. Introduction

The United States of America (US) and the Peoples' Republic of China (PRC) are the world's two largest economies, with 2018 Gross Domestic Products (GDP) of 20.5 and 13.6 trillion United States dollars (USD) respectively, according to a study done by the World Bank [1]. The afore-mentioned values represent 39.7% of the total global GDP.

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According to the Office of United States Trade Representative [2], exchange of goods and services between the two countries had a 2018 value of 737.1 billion USD which represents nearly 1% of total global GDP. As of 2018, the trade deficit between the two nations sits at 419.2 billion USD in favor of the PRC. Considering the size and the importance of this trade relationship, economic cooperation between the two powers is of vital importance to overall global prosperity, however in recent years relations home come to an impasse. In addition to the damage being done to US-SINO relations, the "trade war" has strong implications for all world economies as the sum of these two nations represents nearly 40% of the world GDP, and as such, it is implicit that any damage done to their economic growth significantly affects world growth.

As the dispute intensifies it is important to consider the economic "collateral damage" done to third parties, who have little or no interest or in or ability to effect outcomes of this dispute yet suffer a large portion of the brunt of this ordeal. To date, a multitude of studies have been carried out to assess the damage to both the US and Chinese economies, yet little or no time has been spent assessing the implications and or impacts on small third-party economies.

In the context of a vast and highly interconnected globalized world economy, governments should carefully consider the implications of their policy decisions weighing not only the direct impacts but also the magnitude of the indirect consequences. As such this study will attempt to assess the impact of the trade war to date on Chile is an ideal candidate considering its strong trade ties to both the US and PRC, and its percentage of trade to GDP which sits very close to the world average, (55.7%) according to a study carried out by the OECD [3] as well as having strong market ties to both of these countries, making it a good proxy for understanding the effects of the trade war on other regions.

The main goal of this article is to analyze how a third country is indirectly affected by the problems that the two main economies of the world may have. Here we are using the main stock market indicator of the Chilean Stock Exchange as a variable of such analysis. The manuscript is organized as follows: In the next Subsections 1.1 and 1.2, we discuss studies on the effects of economic protectionism and some previous methodologies applied to measuring the impacts of trade wars. In Section 2, we introduce the data set and methodologies. In Section 3, we analyze the CAR value events, we test also for normality by utilizing a robust class of tests. In Section 4, we introduce a novel model for changes in trade volumes.

1.1. Effects of economic protectionism

Protectionism or the act of seeking to positively influence domestic production and economic performance through the use of government, or other regulations to restrict imports has been around for several centuries at a minimum [4]. These policies have their most recent and more informal origins in 1800s French and English mercantilism. Notwithstanding, to this day there is still significant debate as to whether protectionism is a driver of economic welfare loss, and in the case that it is, to what extent. Another area of debate is who ultimately pays the price of tariffs and other measures.

[5] suggested that the costs of economic protectionism are not as severe as generally agreed upon. This article suggests that some of the economic turbulence generally attributed to protectionism such as the depressions of the 1930s and 1980 can actually

be traced to financial instability and credit difficulties. It is the view of this article that the idea of protectionism as a driver of major economic instability is essentially a myth.

[6], however, took a much more severe stance on the effect of protectionism citing a rise in protectionist measures as one of the principal motors driving the great depression. [7] examined trade tendencies and protectionism in the between-war period of the 1930s and concluded that protectionist measures resulted in a dramatic drop in overall trade, a drastic shortening of average route length, and the strengthening of politico-economic trade blocks. This work seems to agree that protectionism has dramatic economic consequences, as well as driving the intensification of political tensions.

[8] examined the specific historical effect of sanctions from a U.S. standpoint analyzing their effect on the rest of the world. It was concluded that U.S. sanctions can be very damaging to foreign economies, specifically considering that they tend to be targeted on specific countries and industries, and as such are capable of causing serious consequences. This article also concludes that free trade associations tend to promote trade within themselves, while also having the consequence of dampening trade between unaffiliated regions. [4] examined protectionism from 4 distinct viewpoints, and ultimately concludes that these practices are quite beneficial to the specific sector of the economy to which imports are restricted, however, they are a significant driver of overall welfare loss.

As would be expected, non-entry trade measures seem to be more effective than tariffs which allow goods to be imported with a specific tax on the item, making them less competitive in the market. [9] studied the effect of inefficient entry on local production costs and found that both average production costs and price charges increased in the industry are protected. There was a general loss of welfare and consumers paid the costs.

Considering the sheer volume of literature supporting the idea that protectionism is either a driver or a major driver of overall welfare and value loss it seems necessary to conclude that this may be the case.

1.2. Previous methodologies applied to measuring the impacts of trade wars and other policy shocks on economies and markets

Addressing the specific consequences of the current trade war [10] used a descriptive methodology comparing the preparedness of the U.S. and China to handle a trade war and then examine existing economic data to assess impacts. It was concluded that the U.S. is better prepared to confront this challenge as far less of its economy relies on Chinese trade both in terms of USD value and % of GDP. It also concluded that the U.S. has seen far less economic hardship as a result and it is well positioned to continue and leverage its position for a good outcome.

The preferred method of estimating the economic and welfare loss created by the trade war appears to be via computed general equilibrium models or (CGE). Over the short course of the trade war, a considerable amount of the literature estimating its effects using CGE models has been created. [11] attempted to apply one such model to estimate impact on U.S., Chinese, Australian, and overall world economies. This work used the Global Trade Analysis Project (GTAP) CGE model with some modifications and found that the joint effect of trade tariffs (U.S. and Chinese) had an overall negative effect on GDP, employment and consumption in both China and the U.S. but surprisingly had an overall positive effect on Australian GDP and consumption while employment figures remained unchanged. It is questionable whether GDP is really a good indicator of welfare. For further references see [12] or [13]. The authors suggest a new index for measuring welfare based on the human-scale-development approach [12] quantifying the subjective perception of the satisfaction of fundamental human needs.

[14] also used a CGE model, in this case, a static CGE model to estimate the level of welfare loss caused. His finding was consistent with the majority of studies cited in this literature review, as it concluded that both countries suffered significant welfare losses with China losing significantly more than the U.S. [15] also attempts to estimate welfare loss using a CGE model with data from the GTAP 9.0 database, and had similar findings, that is to say considerable welfare loss to both economies with a more significant decline in China.

[16] used a novel partial equilibrium model to estimate the future effects of the trade war on both Chinese and east Asian economies in general. His paper focused on identifying products that could potentially be substituted by competing economies in the region. He concluded that considering tariff levels at the time of publishing there would be a 0.3% drop in Chinese GDP. This article also estimates that the impact on GDP in many potential substitute markets which would look to replace Chinese production could be significant, with Vietnam, The Philippines, and Cambodia leading the way. In total eight countries in the region could experience GDP growth of over 1% as a result of continued tensions.

Shifting this examination to the subject of market impacts [17] examined the market value of firms in relation to their exposure to global value chains involving the U.S. and Chinese firms. Their work analyzed the immediate impact on stock prices of firms in a reduced time frame, and then compared real-world results to estimated results based on the idea that firms with greater exposure to global value changes would incorporate these policy shocks into their market valuation. The finding of their paper was that real-world results closely followed expected results and that U.S. firms with elevated exposure to Chinese supply chains were the most severely affected.

Many authors such as [18–21] have applied methodologies of data analytics involving abnormal returns and cumulative abnormal returns in a specific event window on capital markets in order to assess the impact of policy shocks and other types of events to make judgments on the type and magnitude of the market's reaction; see Figures 1 and 2 for cumulative abnormal returns of specific events. This methodology is particularly useful in assessing such impacts as it can track the outcome of multiple events or announcements and make a cumulative judgment on the overall outcome. This type of work makes use of portfolio theory first developed by [22] which calculates an expected rate of return for a financial instrument based in its systematic and nonsystematic rate of risk. Real-world returns are then compared with expected returns within a specific event window in order to assess the impact of events within that event window.

2. Materials and methods

2.1. Data Description

An initial review of the principal events that composed the trade war revealed 29 significant dates between its origins in June of 2016 when US President announced he would apply tariffs under sections 201 and 301 of the 1974 Trade Act, through October

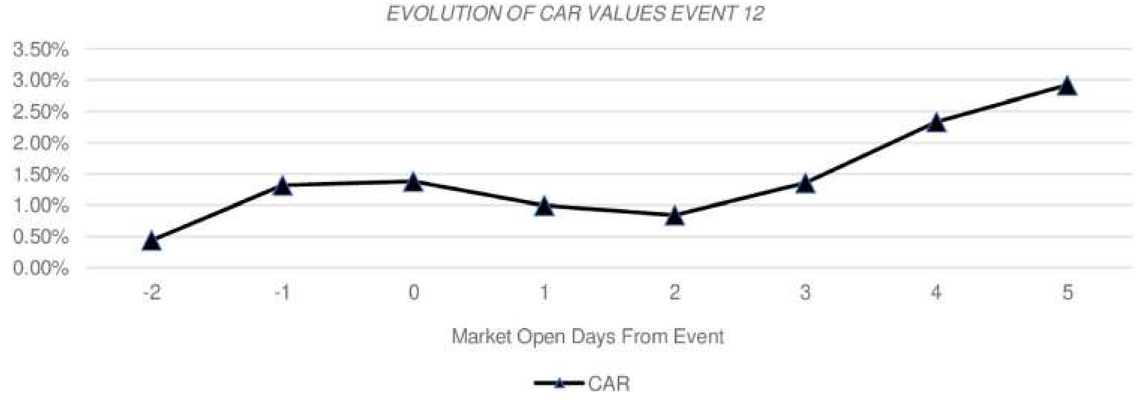


Figure 1. Cumulative abnormal returns for event 12, Source: Santiago Stock Exchange, Authors' Calculations.

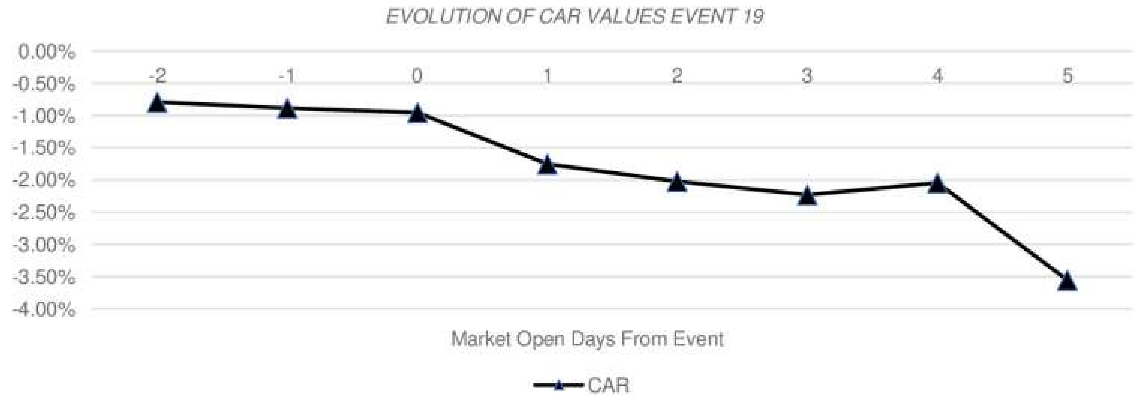


Figure 2. Cumulative abnormal returns for event 19, Source: Santiago Stock Exchange, Authors' Calculations.

of 2019 when phase one of the trade deal would suspend further tariffs. These events were determined by [23]. As a matter of judgment, this study has suspended all events with a date after September 10^{th} of 2019 due to significant unrelated market volatility connected to local protests, leaving a total of 26 (3 events were discarded as a result of this decision) events through August 2019.

Having chosen a finite number of events this study compiled data for market performance from the IPSA (Chiles principal stock index) for the aforementioned dates; see Table 1 for expected returns. The values obtained were retrieved from the Santiago Stock Exchange (Bolsa de Comercio de Santiago). The data points used were once daily closing values for the event date as well as five market open days after and 2 days before as well as an additional 60 market open days before. This allows for 60 values for the calculation of $E[R]$ (expected return) as well as an 8-day event window. In the event of the market being closed on the day of the event, the next market open day was taken as the event day. A total of 1768 data points were used, although some were repeat values as there was some overlap in event windows. Over the entire data set, the highest daily increase was 6.9%, the lowest daily drop was -5.86% and the average daily return was -0.01%.

2.2. Methodology

The Mean Adjusted Model Method for event studies was used to calculate $E[R]$, essentially a 60-day pre-event average. This decision was made as a result of other methods

Table 1. Expected returns for selected events. Source: Santiago Stock Exchange, Authors' Calculations.

Number	Event date	$E[R]$
1	28/6/2016	0.02%
2	31/3/2017	0.27%
3	6/4/2017	0.23%
4	19/7/2017	0.07%
5	14/8/2017	0.08%
6	22/1/2018	0.09%
7	8/3/2018	0.22%
8	2/4/2018	−0.04%
9	3/4/2018	−0.04%
10	4/4/2018	−0.03%
11	15/6/2018	−0.01%
12	10/7/2018	−0.10%
13	1/8/2018	−0.10%
14	7/8/2018	−0.08%
15	24/9/2018	−0.02%
16	1/12/2018	−0.06%
17	1/5/2019	−0.08%
18	3/5/2019	−0.09%
19	5/5/2019	−0.09%
20	16/5/2019	−0.14%
21	18/6/2019	−0.07%
22	29/6/2019	−0.05%
23	1/8/2019	−0.04%
24	5/8/2019	−0.03%
25	13/8/2019	−0.03%
26	23/8/2019	−0.02%

relying on regression and the calculation of slope, intercept, α, and β, and this is inappropriate when working with a stock index as opposed to a specific stock. $E[R]$ was calculated separately for each event resulting in 26 distinct values. Having compiled the necessary data and calculated expected returns for the period in question this study proceeded to perform statistical analysis. A variant of a methodology commonly used in event studies such as [18–21] was used. This methodology was selected because it is very common in literature for event studies. This methodology determines abnormal returns, cumulative abnormal returns, and average cumulative abnormal returns in order to conclude the magnitude and nature of the effect of an event on market values (negative or positive). This analysis determined the average cumulative abnormal return for each event using the following methodology:

$$\mathrm{AR}_{it} = R_{it} - E[R_{it}],$$

where AR_{it} = abnormal return i on day t of the event window;

$$\mathrm{CAR}_t = \sum_{i=1}^{n} \mathrm{AR}_{it},$$

where CAR_t = cumulative abnormal return for the event;

$$\mathrm{CAAR}_t = \frac{\sum_{i=1}^{n} \mathrm{AR}_{it}}{n},$$

where CAAR_t = cumulative average abnormal return for the event.

CAR values below zero indicate that an event has caused a negative impact, whereas a value over zero would indicate a positive impact. The larger this value, the more significant the impact of the event. Values closer to zero indicate that independent of any daily volatility in AR, over the course of the event the impact was neutral.

Additionally, an estimation of monetary variation was made by assessing the impact of CAR for each event on the overall market capitalization of the IPSA at that time. As no historic information of market capitalization was available, but IPSA values were readily available an effort was made to discern the exact relationship between IPSA values and the overall market capitalization of the index. Both IPSA and market capitalization values were registered for several days and linear regression was used to determine the relationship between the two. As the two had a strong correlation ($R^2 = 0.9244$) these values were then used to determine an estimated value for each event ($Y = 27465X - 764032$ with Y being estimated market cap and X being the current IPSA value) which could then be employed in conjuncture with each event CAR value to determine the overall market capital gain or loss as a result.

For the purposes of this study, a comparison of CAAR before and after the event window was used to determine the market reaction. A paired t-test of two samples for means was conducted to test whether there were statistically significant differences in CAR before and after trade war events, and if so what type of reaction had been elicited (positive or negative). t-tests are generally used to determine that the chance data are behaving randomly within normal behavior or if a specific event has altered its behavior. Generally, a P value below 0.05 signifies a statistically significant event. The t-test considered an α of 0.05.

The analysis considered a period of 2 market open days before and 5 market open days after each event in addition to the event day itself, as evidence exists that considering longer periods of time tends to create bias in results as shown in [24]. We calculated the cumulative average abnormal return for a statistically representative population of events within the overall framework of the trade war, and outlined some macro impact of the trade war on Chilean equity markets.

3. Results and analysis

From this point forward each event will be referred to as a number between 1 and 26 according to their date of occurrence with the most recent events having the highest values. For a full table description of events selected see annexed documents. The expected returns for each of the events being studied were calculated and the results were as follows:

Having calculated $E[R]$, AR and CAR were now calculated. A sample of 3 events was graphed in order to illustrate the effect of the event on CAR over the course of the time frame studied.

In the case of event 1 CAR drops strongly in the days before the event, and then makes a strong comeback before finishing the period with a slightly positive CAR at 0.44%. This would indicate that event 1 of the trade war provoked a slightly positive reaction from the IPSA index.

In the case of Event 12, the event begins with a strongly positive reaction in CAR, which dips in the days directly after the announcement but then returns in the last few

Table 2. Final CAR values for each event period. Source: Santiago Stock Exchange, Authors' calculations.

Event	CAR at event window end
1	0.44%
2	−1.71%
3	−0.49%
4	0.28%
5	−0.08%
6	0.89%
7	−0.87%
8	3.09%
9	2.89%
10	1.50%
11	−2.97%
12	2.92%
13	−0.16%
14	−1.51%
15	−0.88%
16	−1.09%
17	−2.19%
18	−1.57%
19	−3.56%
20	−0.67%
21	1.12%
22	0.12%
23	−1.43%
24	−2.61%
25	−2.34%
26	0.82%

days, finishing with a CAR of 2.92%. This would indicate that event 1 of the trade war provoked a strongly positive reaction from the IPSA index.

In the case of event 19, the period began with a strong downturn which intensified over the course of the timeframe culminating in a CAR value of −3.56%. This would indicate that event 19 of the trade war provoked a strongly negative reaction from the IPSA index. CAR values for individual periods are given in Table 2.

A simple analysis of CAR value results for the last day of each event without considering statistical significance or pre-event tendency reveals 16 negative market reactions as well as 10 positive reactions. This would indicate that although some events were processed positively by the market, but the overall reaction was negative, with an aggregate value of -10.04%.

Applying the percentual variation in CAR for each event to the total market capitalization at that time permitted the estimation of total IPSA market capitalization loss or gain as a result of the trade war. Variations in market Capitalization are presented in Table 3. Event impacts on CAAR pre- and post-event are in Table 4.

The Cumulative effect on market capitalization was determined to be a net negative equivalent to a loss of approximately 13 billion USD in market valuation.

Having calculated both AR and CAR for each of the events, a paired *t*-test of two samples for means was conducted in order to determine the impact of the event (negative or positive) as well as whether the event provoked a statistically significant response in markets.

Upon examination of the two-tail paired *t*-test of two samples for means and the difference in CAAR values pre and during the events it was concluded that 18 of 26 events

Table 3. Variation in market capitalization. Source: Santiago Stock Exchange, Authors' Calculations.

Event	IPSA	Market Cap. thousands USD	Event CAR	Variation thousands USD
1	3936	$107.339.032	0.4%	$474.624
2	4783	$130.612.598	−1.7%	($2.231.554)
3	4898	$133.749.651	−0.5%	($652.007)
4	5037	$137.585.962	0.3%	$388.299
5	5064	$138.311.312	−0.1%	($110.806)
6	5828	$159.299.516	0.9%	$1.418.488
7	5576	$152.390.970	−0.9%	($1.330.471)
8	5503	$150.364.877	3.1%	$4.642.805
9	5534	$151.238.264	2.9%	$4.368.971
10	5543	$151.468.695	1.5%	$2.279.372
11	5470	$149.478.307	−3.0%	($4.440.479)
12	5323	$145.440.128	2.9%	$4.245.695
13	5398	$147.502.200	−0.2%	($231.364)
14	5328	$145.579.375	−1.5%	($2.192.000)
15	5386	$147.152.845	−0.9%	($1.295.837)
16	5152	$140.726.585	−1.1%	($1.534.877)
17	5142	$140.449.463	−2.2%	($3.079.299)
18	5132	$140.195.137	−1.6%	($2.194.649)
19	5124	$139.972.396	−3.6%	($4.976.034)
20	4978	$135.949.872	−0.7%	($915.888)
21	5041	$137.675.223	1.1%	$1.540.743
22	5063	$138.288.517	0.1%	$160.430
23	4941	$134.935.315	−1.4%	($1.923.420)
24	4780	$130.529.379	−2.6%	($3.412.119)
25	4846	$132.320.372	−2.3%	($3.091.626)
26	4649	$126.910.316	0.8%	$1.046.308

Table 4. Event impact on CAAR pre- and post-event. Source: Santiago Stock Exchange, Authors' calculations.

Event identifier	CAAR PRE	CAAR EVENT	DIFF.	Reaction
1	0.22%	−0.01%	−0.24%	Negative
2	1.11%	−0.01%	−1.12%	Negative
3	−0.04%	−0.14%	−0.10%	Negative
4	1.19%	0.35%	−0.84%	Negative
5	0.04%	−0.31%	−0.35%	Negative
6	1.12%	0.41%	−0.71%	Negative
7	−2.44%	0.12%	2.56%	Positive
8	−0.97%	1.48%	2.45%	Positive
9	−1.42%	2.06%	3.48%	Positive
10	−1.22%	0.76%	1.98%	Positive
11	1.14%	−1.67%	−2.81%	Negative
12	−1.44%	1.45%	2.89%	Positive
13	1.59%	0.59%	−1.00%	Negative
14	0.50%	−1.06%	−1.56%	Negative
15	1.37%	0.23%	−1.13%	Negative
16	−0.95%	0.38%	1.33%	Positive
17	0.26%	−1.07%	−1.33%	Negative
18	−0.45%	−0.88%	−0.44%	Negative
19	−0.96%	−1.78%	−0.82%	Negative
20	−1.55%	0.07%	1.62%	Positive
21	0.24%	0.07%	−0.17%	Negative
22	0.30%	−0.24%	−0.54%	Negative
23	−0.80%	−2.15%	−1.35%	Negative
24	−1.02%	−2.19%	−1.17%	Negative
25	−2.20%	−1.83%	0.37%	Positive
26	0.53%	−1.77%	−2.30%	Negative

Table 5. *t*-test for determination of significance. Source: Santiago Stock Exchange, Authors' Calculations.

Event identifier	*t*-stat	$P(T \leq t)$ two-tail	*t*-Critical two-tail	Event type
1	0.90	0.399	2.36	Non-sig.
2	3.50	0.010	2.36	Significant
3	0.24	0.818	2.36	Non-sig.
4	1.71	0.130	2.36	Non-sig.
5	1.76	0.122	2.36	Non-sig.
6	4.26	0.004	2.36	Significant
7	−3.52	0.010	2.36	Significant
8	−3.85	0.006	2.36	Significant
9	−6.60	0.000	2.36	Significant
10	−4.25	0.004	2.36	Significant
11	6.02	0.001	2.36	Significant
12	−9.56	0.000	2.36	Significant
13	2.76	0.028	2.36	Significant
14	5.09	0.001	2.36	Significant
15	1.19	0.272	2.36	Non-Sig.
16	−5.43	0.001	2.36	Significant
17	9.52	0.000	2.36	Significant
18	3.31	0.013	2.36	Significant
19	2.98	0.021	2.36	Significant
20	−10.39	0.000	2.36	Significant
21	0.61	0.564	2.36	Non-Sig.
22	2.18	0.065	2.36	Non-Sig.
23	3.19	0.015	2.36	Significant
24	3.20	0.015	2.36	Significant
25	−0.84	0.431	2.36	Non-Sig.
26	3.84	0.006	2.36	Significant

studied provoked statistically significant reactions from the IPSA index, see Table 5. Among those results ($n = 18$) considered to be significant 11 provoked a negative change in CAAR when compared with pre-event CAAR, 7 pushed the market into a positive reaction. Considering the results of CAR, CAAR and the *t*-test it can be concluded that on aggregate the U.S.-China Trade War had a strongly negative effect on the IPSA when compared with periods of normal market performance.

3.1. Robust testing for normality

Distributions of stock market returns are often presented as bell-shaped curves. This representation implies that stock returns are normally distributed, which can depend on the period analyzed and the frequency of sampling prices to calculate returns. For e.g. a return distribution that contains returns realized during the financial crisis will be very different than one covering a different period. However, expected returns in our case are far from the normal distribution, mainly due to the presence of outliers – see the first boxplot and histogram in Figure 3.

For the purpose of testing for normality of the above-presented data sets, we use selected classical tests for normality as well as selected robust normality test (see [25]). So, in total, we used four classical tests for normality – Anderson-Darling (AD) test, Jarque-Bera (JB) test, Lilliefors (LT) test, and Shapiro–Wilk (SW) test. For the purpose of comparison, we also used four robust tests for normality – robust Jarque-Bera (RJB) test, medcouple (MCLR) test, and two variants of the RT class tests introduced by [25].

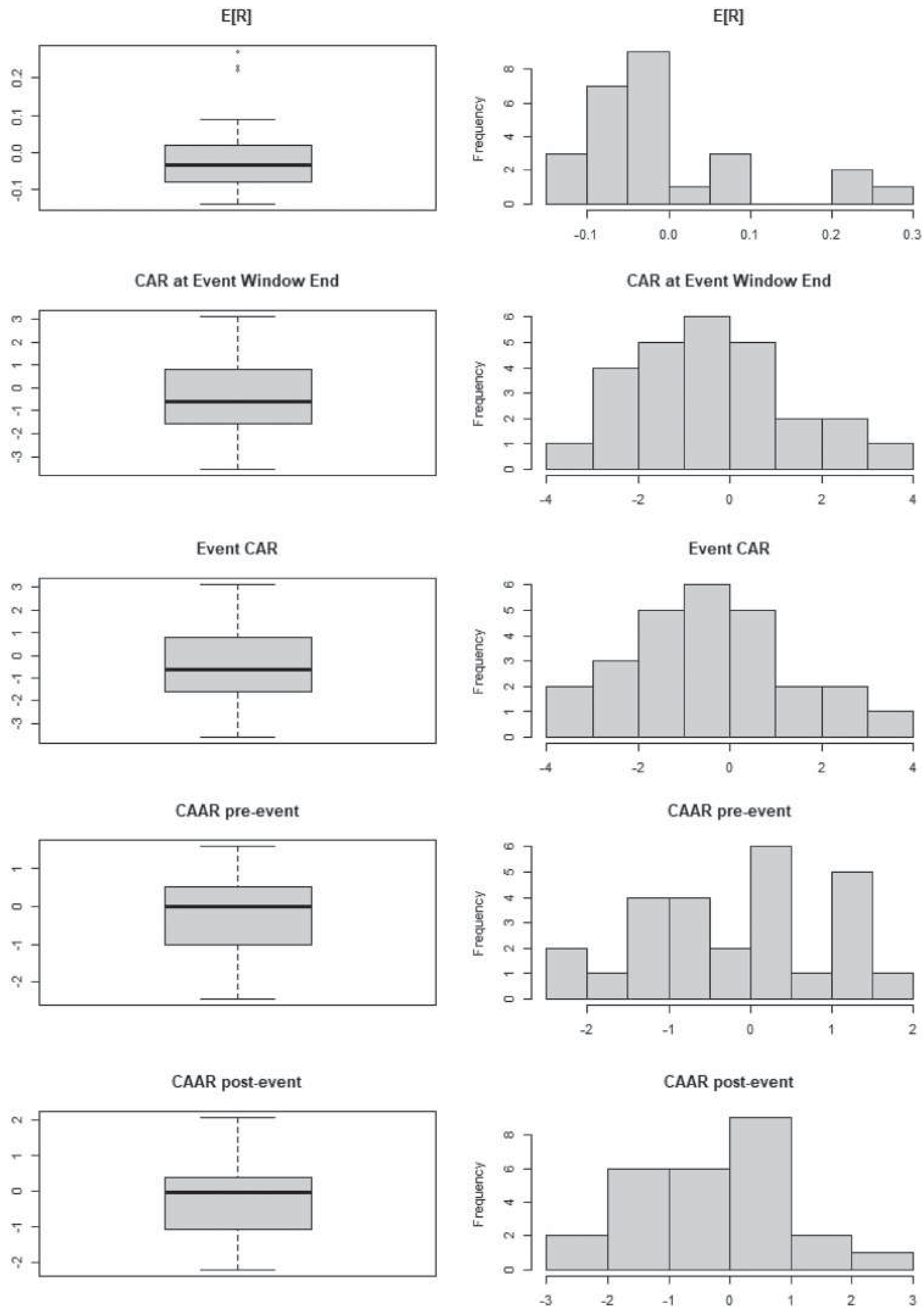


Figure 3. Boxplot and Histograms for the analyzed data sets.

The descriptive statistics are presented in Table 6 and boxplots and histograms are in Figure 3. Only the first dataset, expected return E[R] is characterized by significant asymmetry, above-normal kurtosis, and the presence of outliers. Therefore, all tests reject the null hypothesis of normality of distribution, at a 5% significance level – see

Table 6. Descriptive statistics for the analyzed data sets.

	Min	Median	Mean	Max	SD	Skewness	Kurtosis
E[R]	−0.14	−0.04	−0.01	0.27	0.11	1.38	4.03
CAR at event window end	−3.56	−0.58	−0.39	3.09	1.78	0.34	2.50
Event CAR	−3.60	−0.60	−0.40	3.10	1.78	0.35	2.52
CAAR pre-event	−2.44	0.00	−0.23	1.59	1.14	−0.16	1.99
CAAR post-event	−2.19	−0.01	−0.27	2.06	1.15	−0.05	2.30

Table 7. Results of testing for normality for the analyzed data sets, 1 = CAR at Ev. Wdw. End, 2 = Ev. CAR, 3 = CAAR pre-event, 4 = CAAR post-event.

test	E[R]		1		2		3		4	
	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
AD	1.768	0.000	0.223	0.814	0.226	0.804	0.429	0.293	0.539	0.154
JB	9.430	0.013	0.783	0.572	0.770	0.579	1.207	0.363	0.539	0.711
LT	0.248	0.000	0.082	0.922	0.083	0.916	0.122	0.395	0.142	0.191
RJB	32.977	0.005	0.632	0.618	0.629	0.620	1.030	0.399	0.139	0.923
SW	0.825	0.000	0.970	0.618	0.970	0.627	0.955	0.311	0.951	0.247
MCLR	6.801	0.047	0.492	0.910	0.484	0.912	3.674	0.234	3.995	0.200
MMRT1	8.524	0.011	0.931	0.517	0.952	0.508	1.955	0.218	1.378	0.352
MMRT2	6.055	0.015	0.977	0.507	0.998	0.498	2.216	0.180	1.665	0.283

Table 7 for p-values of tests for normality for the analyzed datasets. However, at the 1% significance level, some tests do not reject the null hypothesis – specifically the Jarque–Bera test, the medcouple test, MMR1, and MMRT2 tests that are more robust than other tests.

4. Modeling of changes in trade volumes

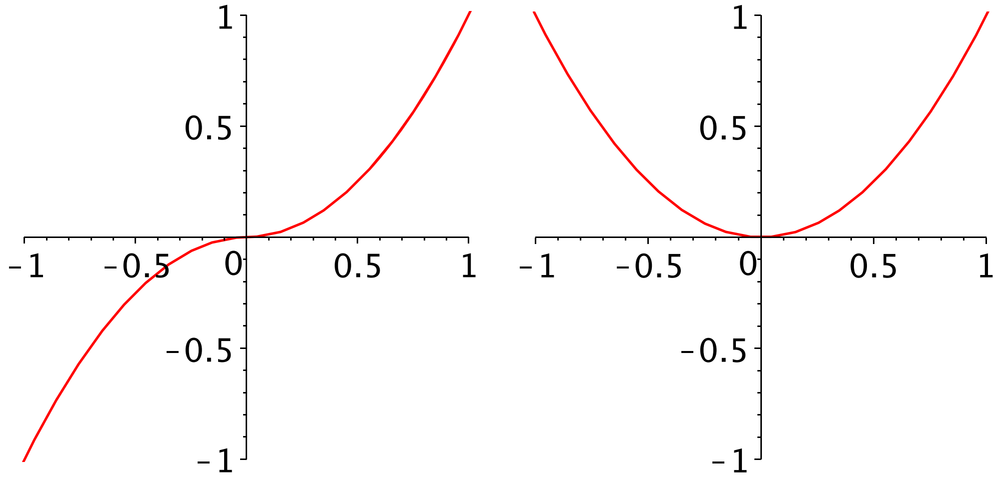
In this section, using the delta method constructed confidence interval, we will show significant statistical differences between trading volumes measured by IPSA before and after the change point. Consider a probability space $(\Omega, \mathcal{F}, P)$ and a measurable space (S, Σ), on which a stochastic process lives, i.e. a collection of S-valued random variables, which can be written as $\{r(t, \omega) : t \in T\}$ (to reflect that it is actually a function of two variables). We usually use shorten notation r_t.

Assume that IPSA r_t evolves as a real-valued stochastic process. Note that at each point in time, the expectation $m(t) := E[r_t]$ of the random variable is the mean (we assume here only L^2 processes). Thus, the mean and also variance $w(t) = E[r_t^2] - E[r_t]^2$, in general, is a function of time.

We now define a new statistical quantity, aggregated IPSA for $u, v \in T, u < v$

$$RC(u, v) := \int_u^v m(t)\ \mathrm{d}t \tag{1}$$

In addition to classical power function $x^k, x \in \mathbb{R}, k \in \mathbb{N},$ we use the notation $x^{k^*} := |x|^k \mathrm{sgn}(x), x \in \mathbb{R}$ is a signed power function which guarantees that power $k \geq 0$ might be real for any real values of x. Clearly, functions x^m and x^{m^*} are different for negative values, since x^{m^*} is odd, see e.g. case $m = 2$ in Figure 4. Suppose that $r_0 = A, r_0' = B \in \mathbb{R}$ and σ is nonnegative. For intern local dynamics (of IPSA), as will be clear later, we use Chan–Karolyi–Longstaff–Sanders model (1992) with a fixed parameter k



(a) $x^{2^*} = |x|x$ (b) x^2

Figure 4. Graphs of power functions on $[-1, 1]$. (a) $x^{2^*} = |x|x$ and (b) x^2.

$$\mathrm{d}p_t = (\theta - \beta p_t)\ \mathrm{d}t + p_t^k\ \sigma\ \mathrm{d}W_t \tag{2}$$

For convergence of interest rate models of this type see [26].

Note that (2) involves several known processes. The Cox–Ingersoll–Ross model supposes $k = 1/2$, the Geometric Brownian motion model supposes $k = 1$ and $k = 0$ implies the famous Ornstein–Uhlenbeck model or the Vasicek model. We focus later here on the last case.

Note that the stock market index, is an index that measures a stock market, or a subset of the stock market, that helps investors compare current stock price levels with past prices to calculate market performance. Typically stock market dynamics are modeled by a Stochastic Differential Equation. Here we consider that the IPSA is driven by the following two-dimensional IVP (two-factor model), where (2) is explicitly included for $a = -\beta$ and $b = \theta$. For a better understanding and its good properties see [27] or [28].

$$\begin{aligned} \mathrm{d}r_t &= \left[f\ r_t + c\ p_t^{m^*} + e\right]\mathrm{d}t, \\ \mathrm{d}p_t &= [a\ p_t + b]\mathrm{d}t + \sigma\ p_t^k\ \mathrm{d}W_t, \\ r_{t_0} &= A,\ \ r'_{t_0} = B. \end{aligned} \tag{3}$$

This model is unique in the way of raising power by p_t. For classical power p_t^m and more general settings see e.g. [27] or [28]. Note that by changing the assumption of raising power three situations could arise. The solution (with common values of all parameters) could coincide, could coincide only for a specific interval, or could be different on the whole interval of existence, see [27]. Note that the value of m is in the role of stabilization of the process's speed and may influence the value of σ, which is a very interesting fact.

Now, for simplicity, we suppose that $k = 0$, and we are also forced to consider that $c \neq 0$. We obtain specific time-integral of signed powered Ornstein–Uhlenbeck process. The system (3) reduces to the nonlinear model, in which r_t can be found explicitly, see [27] or [28], e.g. for $t_0 = 0$ we have $r_t = \mathrm{e}^{ft} \int_0^t \mathrm{e}^{-fs}((\sigma\ \int_0^s \mathrm{e}^{-av}\ \mathrm{d}W_v + \frac{b(1-\mathrm{e}^{-as})}{a} + \beta_m)^{m^*} \mathrm{e}^{mas}\ c + e)\ \mathrm{d}s + A\mathrm{e}^{ft}$, where $\beta_m = \left(\frac{B}{c}\right)^{\frac{1}{m}^*}$.

We also assume that $f=0$ and $m=1$ are in order to obtain estimable parameters problems. Note also that the proper estimation of m is quite a difficult open problem. In addition, in the case when $m=1$, the model (3) reduces to the following system

$$\begin{aligned} dr_t &= [c\ p_t + e]dt, \\ dp_t &= [a\ p_t + b]dt + \sigma\ dW_t, \\ r_{t_0} &= A,\ \ r'_{t_0} = B. \end{aligned} \tag{4}$$

It is a special case of the linear stochastic equation (for more details see [29]):

$$\begin{aligned} d\mathbf{X}_t &= (\mathbf{A}(t)\ \mathbf{X}_t + \mathbf{a}(t))\ dt + \boldsymbol{\sigma}(t)\ d\mathbf{W}_t, \quad t_0 \le t < \infty, \\ \mathbf{X}_{t_0} &= \xi, \end{aligned} \tag{5}$$

where $d \times d, d \times 1$ and $d \times r$ matrices (in our case $d=2$ and $r=2$) $\mathbf{A}(t), \mathbf{a}(t)$ and $\boldsymbol{\sigma}(t)$ are nonrandom, measurable, and locally bounded, whereas one can obtain an explicit solution in the form

$$\mathbf{X}_t = \boldsymbol{\Phi}(t)\left[\boldsymbol{\Phi}(t_0)^{-1}\xi + \int_{t_0}^{t} \boldsymbol{\Phi}(s)^{-1}\ \mathbf{a}(s)\ ds + \int_{t_0}^{t} \boldsymbol{\Phi}(s)^{-1}\ \boldsymbol{\sigma}(s)\ d\mathbf{W}_s\right], \tag{6}$$

where $\boldsymbol{\Phi}$ is a fundamental matrix, i.e. the matrix solution of the problem $\boldsymbol{\Phi}(t)' = \mathbf{A}(t)\boldsymbol{\Phi}(t)$. Clearly

$$\mathbf{m}(t) := E[\mathbf{X}_t] = \boldsymbol{\Phi}(t)\left[\boldsymbol{\Phi}(t_0)^{-1}\xi + \int_{t_0}^{t} \boldsymbol{\Phi}(s)^{-1}\ \mathbf{a}(s)\ ds\right] \tag{7}$$

4.1. Fitting the model

Here, we focus on parameters of the model (4). Since it is a special case of (5) we have $\xi = (A, B), \boldsymbol{a}(t) = (e, b), \boldsymbol{\sigma}(t) = (0, \sigma)$ and $\boldsymbol{\Phi}(t) = \begin{pmatrix} 1 & \frac{c}{a}e^{at} \\ 0 & e^{at} \end{pmatrix}$, and from (6) and the first line in (7) we have

$$r_t = m(t) + \frac{c\sigma}{a}\int_{t_0}^{t} (e^{a(t-s)} - 1)\ dW_s$$

and

$$m(t) = \frac{c(Ba+b)e^{a(t-t_0)} + (e(t-t_0)+A)a^2 - ((t-t_0)b+B)ca - bc}{a^2} \tag{8}$$

Note that for $a \to 0$ we have from (8) that $m(t) \to (Bc+e)(t-t_0) + A + \frac{bct^2}{2} - t_0bct + \frac{bct_0}{2^2}$ and for $c \to 0$ that $m(t) \to e(t-t_0) + A$. Variance can be easily computed by using Ito isometry on $E[r_t^2]$ yielding

$$w(t) = \frac{c^2\sigma^2}{a^2}\int_{t_0}^{t} (e^{a(t-s)} - 1)^2\ ds = \frac{c^2\left(e^{2a(t-t_0)}/2 - 2\ e^{a(t-t_0)} + 3/2 + a(t-t_0)\right)\sigma^2}{a^3} \tag{9}$$

with $w(t) \to c^2\sigma^2(1/3\ t^3 - t^2t_0 + t\ t_0{}^2 - 1/3\ t_0{}^3)$, if $a \to 0$ and $w(t) \to 0$, if $c \to 0$.

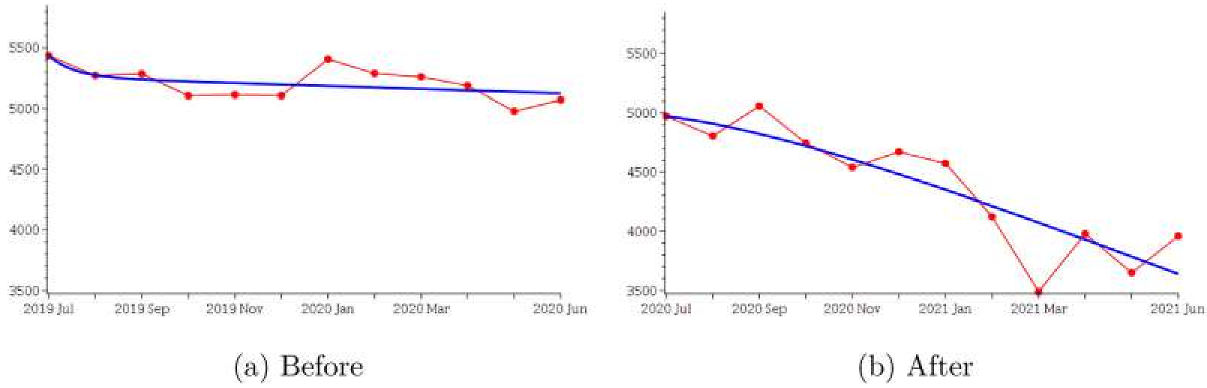


Figure 5. Fitting of $m(t)$ on two real data periods. (a) Before and (b) after.

We have fixed the time-change point as 42 due to a 6-month delay due to the COVID period, thus we have two problems with different estimated parameters (we have used (8) and for given experimental data procedure `NonlinearFit()` from package `Statistics` in software `Maple 2019`):

(i) $t_0 = 31, A = r_{31} = 5434.44, B = r_{32} - r_{31} = -164.00 \Rightarrow \hat{a} = -1.800, \hat{b} = -14.858, \hat{c} = 2.052, \hat{e} = 4.847$

(ii) $t_0 = 43, A = r_{43} = 4972.36, B = r_{44} - r_{43} = -167.99 \Rightarrow \hat{a} = -0.296, \hat{b} = -124.795, \hat{c} = 0.399, \hat{e} = 17.216$

In Figure 5 one can see the situation before and after the threshold time-point with values from i) and ii) respectively.

Now, based on the data-driven confidence intervals approach for diffusion processes [30] and nonparametric delta method [31], we consider $100(1-\alpha)\%$ asymptotic normal confidence interval of the form

$$\left(RC(u,v) - z_{1-\alpha/2}\sqrt{V(u,v)},\ RC(u,v) + z_{1-\alpha/2}\sqrt{V(u,v)}\right), \tag{10}$$

where $V(u,v) = \int_u^v w(t)\, dt$ is the aggregated variance. We also assume that σ is for both periods in i) and ii) equal to 1. See also Figure 6 where variances given by (9) with estimated parameters from i) and ii) are plotted. Note also that in the definition of RC and V can time averaging be used by dividing it by the interval width. In our setup and for $\alpha = 0.05$, we receive two non-overlapping confidence intervals, for choice $z_{1-\alpha/2} = 1.96$. Namely, these intervals are $(57176.91, 57209.10)$ before, and $(48184.98, 48210.48)$ after the change-point time.

Note that estimation of parameters a, b and thus also of σ can be obtained from data $p_{t_i} = \frac{r_{t_{i+1}} - r_{t_i}}{\delta}$ (in the sense of discretization with suitable positive δ) by MLE or OLS for OU process, see e.g. [32].

5. Discussion and conclusions

As resumed in Subsection 1.1, economic theory lets little doubt that protectionism and trade barriers can negatively affect *allocative efficiency* and hence provoke overall welfare losses. This article contributes to the discussion in economic theory giving evidence that

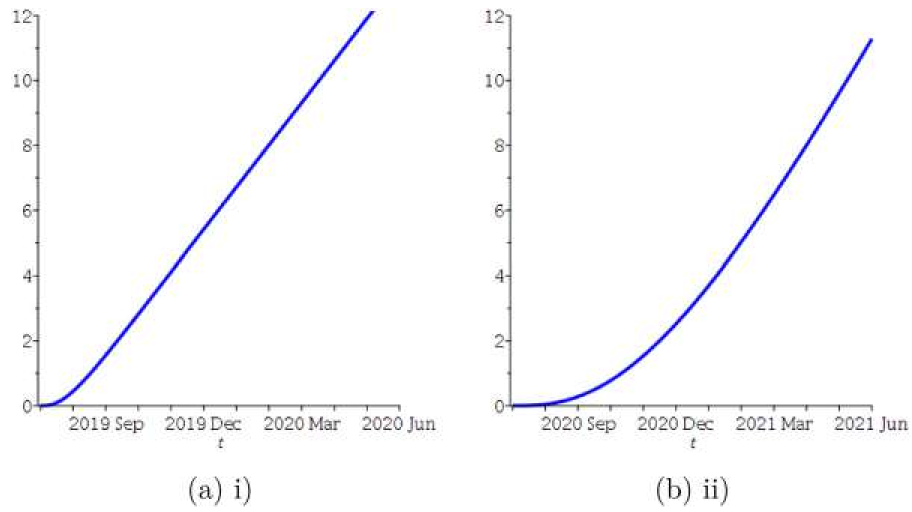


Figure 6. Graphs of $w(t)$ for two different periods. (a) (i) and (b) (ii).

welfare losses do not only occur in the economies that are the targets of the protectionist measure but even in third economies, which are not directly part of the 'trade war', here Chile.

However, the impact of the US-Chinese trade war on the stock markets in Chile (and probably elsewhere) is not necessarily at all times the same. The world financial system sliders into crisis at regular intervals. Evidence exists that the long economic waves, the so-called Kondratieff waves, are, in fact, cycles of the financial system [33,34]. Symptoms that indicate that we are close to the next mega-crisis, comparable to the Great Depression, are money supply and debt having reached unprecedented levels in all industrialized economies and, associated herewith, the increasing number of credit defaults and speculative bubbles on stock and real estate markets (as well as other perceived 'save havens' such as gold or cryptocurrencies).

The reason why the financial system falls into deep crisis at regular intervals is unfortunately not well understood in economics. A cause could be the unnatural design of our money, see [35] and [13]. The closer we get to the inevitable collapse [36] and the bigger the price bubbles on stocks and other investment markets already are, the higher the probability that markets can be impacted by negative trade signals such as trade wars. Hence, it is not necessarily protectionism that causes the statistically significant market reactions, but these could be merely triggered by such events. On the other hand, the closer the world financial system gets to collapse, the more nervous become politicians and it is more likely that protectionist measures will be applied. This could explain the positive correlation between protectionism and the Great Depression authors [6] and [11] mentioned. It would be interesting to conduct the same study after the next crisis and the reset of the financial system, i.e., when money supply is still low, and we will consequently see less volatility and only little speculative bubbles on investment markets. Likely, a trade war would then have less impact on the expected returns of stock markets.

This study attempts to determine the impact of policy announcements in the U.S. Sino trade war between 2016 and 2019 on Chilean equity markets. The mean adjusted model method for event studies was used to determine expected returns and a CAR and CAAR based methodology comparing values within and outside of event windows was used to determine market reactions. This analysis showed a strong negative reaction to the trade war in overall terms. A two-tail paired *t*-test of two samples for means was employed to discern if the market reaction was statistically significant, and in most cases, it was (18 of 26 events). The majority of those events considered significant (11 of 18 significant events) produced negative reactions on the IPSA. The difference between periods is also confirmed by introduced cumulated measures of trade volumes.

We found that the Chilean stock exchange market reacts to the economic war between the US and China. We can of course suggest that this influence on expected returns is because of the trade ties; however, this does not necessarily need to be the case. It could be that investors at the stock exchange are nervous and so any event in the trade war provokes investors to sell stocks in Chile (and elsewhere). The influence on the Chilean market is not necessarily causal to the trade ties. The market, as measured by the IPSA, has been efficient in processing new information in a timely manner. As such local capital markets performed well pricing in the expectations of new volatility in world trade and adjusting equity valuations accordingly.

One of the limitations of this study methodology is that it gauges market valuations based on expectations of economic results, but not the economic results in and of themselves. A possible opportunity to follow up on this study would be employing a CGE or other economic assessment model to measure GDP or other variations. Additionally, studies could be performed to assess variations in firm earnings in relation with the trade war, and additionally compare market reaction with real world earnings.

Here, we used 7 days window, which allows us mainly to concentrate on the immediate reaction of the stock market to the announcement of some information. We do not study possible overreactions of the market and delays. We address partially capital issues, and the relationship between the current and capital account of the balance of payments is not addressed. Instead of this, we show that domestic (Chilean) capital markets were influenced in a certain direction.

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Appendix B

Paper 2

On testing the changes in trends of IPSA and rates

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Abstract

Calibration of interest rate models benefits from grouping data to homogenous classes. Such an approach is typical in many financial time series. Preliminaries have been developed for Cox-Ingersoll-Ross Model models but this issue remains an open problem for many more realistic interest rate models. Here we develop such a strategy for general class interest rate and classes are based on p-value thresholds for testing for normality and gamma distributions. We use as the benchmark financial series of IPSA and its log-returns. We also study the relationship between interest rate and the market returns represented by the IPSA indicator, with positive correlation in some lags which reveals some interesting facts in the contrary to the conventional theory.

Keywords IPSA, interest rate, likelihood ratio test, testing for normality, parameter dependence.

1 Introduction

[Orlando et al. (2019)] provided preliminaries on calibration of Cox-Ingersoll-Ross (CIR) model-based rate by partitions, where individual groups shall follow normal or gamma distribution. This model was introduced by [Cox et al. (1985)] and is known as a one-factor time-homogeneous model, an improvement of the Vašíček model. The CIR model describes interest rate as a diffusion process $r = (r(t))_{t\geq 0}$ unique solution of

$$dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t), \quad r(0) = r_0 > 0,$$

here k, θ and σ are positive constant parameters. Due to mean reversion, as time becomes large, the distribution of future interest rate will approach a gamma distribution. We work here in much more advanced models of interest rate introduced by [Stehlík et al. (2015)], see appendix 1. We consider a problem of testing whether a sample of observations from an interest rate model can be linked to the inter-arrival times given by a counting process. e.g. the counting process coming from Poisson

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distributions with different parameters. Such a problem has been studied by several authors, see e.g. [Brown and Zhao (2002)]. Especially, interest is put to an over-dispersed and under-dispersed data. From the point of view of parsimony, we could think about intensity to be a simple function of time (i.e. piecewise constant intensity function with a finitely many jumps of the selected counting process), $\lambda(t) = \sum_{i=0}^{n} \lambda_i \xi_{[t_i,t_{i+1})}(t)$, here λ_i is a constant value of intensity in the period $[t_i, t_{i+1})$ and $\xi_I(t)$ is a characteristic function of interval I, i.e. for all $t \in I : \xi_I(t) = 1$ and 0 otherwise. To guarantee validity of such an approach in [Stehlík et al. (2018)] we proved the uniform convergence of approximate cumulative distribution functions to the exact distribution functions of exact likelihood ratio test for hypothesis $H_0 : \lambda = \lambda_i$ on each compact set. Thus, it is not necessary to work with arbitrary complex counting processes, if we can calibrate the model of a piecewise constant intensity function given the data. Several special cases can be of interest before the most general model, namely Null Hypothesis may contain of counting process N_i driven by a single Poisson with scale λ. We can opt for testing against two alternatives:
a) general alternative $H_{A1} : X_i \sim Poiss(\lambda_i)$ and scales λ_i are arbitrary.
b) change point alternative $H_{A2} : N_i \sim Poiss(\lambda_i)$ and $\lambda_i = \lambda_1, i \leq k$ and $\lambda_i = \lambda_2, i > k$. Here we consider two cases, namely $\lambda_1 < \lambda_2$ or $\lambda_2 < \lambda_1$. Such concept was applied in [Stehlík (2003)] to receive the decomposition of I-divergence to two LR statistics. Several interesting properties of exact likelihood ratio tests have been observed in [Stehlík and Wagner (2011)], from which, in particular case of our two alternatives (albeit a bit modified), ELR and ELR2 tests are of interest.

Since data of IPSA and/or interest rates are statistically dependent but dependence can be of high complexity, we replace it by parameter dependence models, which are related to development of [Filus et al. (2018)] where relation to series of papers on pseudo-distributions is given. The transformation approach for so called *multivariate pseudonormal distribution* is cited in [Kotz et al. (2000)], pages 217-218. Samuel Kotz acknowledge invariance of pseudonormal distributions with respect to pseudo affine transformation. This was to the best knowledge of the authors the first application of parameter transformation method. The version of method cited in [Kotz et al. (2000)] was defined as follows: Let $T_1, ..., T_k$ be k independent normal random variables. Then one applies the transformation

$$X_1 = aT_1$$

$$X_2 = \phi_1(T_1)T_2 + \theta_1(T_1)$$

$$X_3 = \phi_2(T_1, T_2)T_3 + \theta_2(T_1, T_2)$$

$$X_k = \phi_{k-1}(T_1, ..., T_{k-1})T_k + \theta_{k-1}(T_1, T_2, .., T_{k-1})$$

Here $a \neq 0$ and ϕ_i, θ_i are real continuous parameter functions, assumed to be positive and nondecreasing with respect to each argument. The inverse of this transformation is easily computable and one can get joint density function of $(X_1, ..., X_k)^T$ and conditional density of X_i given $X_1, ..., X_{i-1}$. Any multivariate normal density function is a special case of the above multivariate pseudonormal density when we set ϕ_i to be constant and θ_i to be linear. [Filus and Filus, 2000] and [Filus and Filus, 2001] gave a reliability motivation for pseudonormal distributions. The idea of this modelling can be explained as "parameter dependence". There were gamma-dsitribution related pre-concept to parameter dependence, where the time-to-failure distribution of the system is assumed to be gamma. Both parameters of gamma distribution are continuous functions of the load such that the mean time-to-failure is a decreasing function of the load. The even before, [Filus and Piasecki (1980)] the pre-conception of "parameter dependence" is given.

In this paper we will concentrate on creation of homogeneous data subgroups which can be well used for calibration of IPSA or interest rate. Several alternative approaches could be provided for modeling non-homogeneous Poisson Processes, see e.g. Bayesian approach in [Kuo and Yang (1996)]. Let $y_1,\ y_2,\dots,y_N$ be univariate random interest rates distributed according to the Exponential densities

$$f(y_i|\vartheta) = \lambda_i e^{-\lambda_i y_i}, \text{for } y_i > 0, \tag{1}$$

The basic idea of relating normal or gamma distributed sub-classes to the original counting process is through the parametrization of the intensity, e.g.

$$\lambda_i = \lambda(\mu_i, \sigma_i^2) \tag{2}$$

for Normal (μ_i, σ_i^2) distributed subgroup and suitable function $\lambda : R \times R^+ \to (0, \infty)$ and

$$\lambda_i = \lambda_g(\mu_i, v_i)$$

for each gamma (μ_i, v_i) distributed subgroup and suitable function $\lambda_g : R^+ \times R^+ \to (0, \infty)$. Here we consider mean-parametrization of gamma distribution.

The paper is organized as follows. In the following section we recall tests used in the paper. In section 3 real data illustrations are given. Here we consider IPSA series and its log-returns and empirical dependencies between the parameters are discussed. In section 4 we provide economical and statistical discussion and conclusions. Technicalities are left for the Appendix.

2 Tests for subgrouped data

First, we start with the hypothetical assumption that we can transform our financial data into approximately independent sub-groups. The clustering problem or sub-groups of extensive data are studied in different disciplines and from various points. Mainly, we are interested in financial time series, and we can see the work of [Maharaj and Inder (1999)], [Duncan, Gorr, and Szczypula (2001)], [Baran and Sönmezer (2013)], who assess the forecasting of financial time series, making sub-groups of the data with different approaches. In this section we recall both tests for homogeneity of exponential distribution and robust tests (RT) for normality, which allows us to classify subclasses according to goodness of fit to exponential or normal distributions. The following two subsections are dedicated to these tests.

2.1 Likelihood ratio based tests for subgrouped data

The general subpopulation model which is the alternative tested in the exact likelihood ratio test for homogeneity (ELR) proposed by [Stehlík (2003)] assumes that each observation follows an exponential distribution (1) with its own parameter. A more specific case of a subpopulation model is inhomogeneity with an unobserved clustering and a given number of clusters k, this is the alternative of the exact likelihood ratio test for k subpopulations ELRk introduced in [Stehlík and Ososkov (2003)]. The ELR2 test uses the alternative of two subpopulations, which can be specified by the existence of two nonempty index sets M_1, M_2 such that

$$M_1 \cup M_2 = \{1, ..., N\}, M_1 \cap M_2 = \emptyset \tag{3}$$
$$\forall j \in M_1 : \lambda^j = \lambda^1, \forall j \in M_2 : \lambda^j = \lambda^2, \lambda^1 \neq \lambda^2. \tag{4}$$

Both the mixture and the subpopulation model can be used to model unobserved clustering. In [Stehlík et al. (2018)] we consider a subpopulation of a general sample, in which it is valuable to testify for individual parameter of a given value, namely $H_0 : \lambda = \lambda_0$ versus $\lambda \neq \lambda_0$. Then, exact distribution function and exact density has been derived in [Stehlík (2003)]. The Likelihood ratio tests for the case of Type I and Type II censored data have been studied in [Balakrishan and Stehlík (2014)]. In [Stehlík et al. (2018)] we provided a discussion on three approximations of the exact density: a) by Fourier transformation, b) by saddle-point approximation, c) by approximation driven by numerical solution of ODE, where we prove also uniform convergence of obtained sequence of approximating densities to the exact density. In the next section we provide simulation study for ELR tests.

2.1.1 Simulating powers based on the ELR tests for recognizing the sub-grouping

First, it is necessary to specify the simulation setup:

1. We generate exponential distribution with parameter $\lambda = 1$, i.e. Exp(1) for null hypothesis and computed ELR and ELR2 test statistics for sample sizes $n \in \{10, 20, 50, 100, 200\}$.

2. We found critical constants of ELR and ELR2 test statistics, i.e. 95% quantiles of the distributions of ELR and ELR2 test statistics. These are given in the following table:

Table 1: Critical constants of the ELR and ELR2 test statistics for $\alpha = 0.05$

	$n = 10$	$n = 20$	$n = 50$	$n = 100$	$n = 200$
ELR	9.7839	17.4090	38.1869	70.8623	134.2841
ELR2	6.8426	11.1925	23.2535	42.2952	79.0955

3. As alternative we generate 50% of Exp(1) and 50% of Exp(λ_2) with rate λ_2 where $\lambda_2 \in \{1, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0\}$ and we use critical constants from Table 1 above. We compute power of recognizing non-homogeneity (e.g. of change-point for non-homogeneous-Poisson).

4. Tables 2 and 3 provide sizes and powers of ELR and ELR2 tests

Table 2: Size of the ELR and ELR2 tests

		$n=10$		$n=50$		$n=100$	
λ	mean=$1/\lambda$	ELR	ELR2	ELR	ELR2	ELR	ELR2
1.0	1.000	0.051	**0.050**	**0.050**	0.046	0.053	**0.050**
1.2	0.833	**0.051**	**0.051**	**0.051**	0.053	0.052	**0.049**
1.4	0.714	**0.050**	**0.050**	**0.052**	**0.052**	0.053	**0.052**
1.6	0.625	**0.050**	0.051	0.049	**0.050**	0.055	**0.052**
1.8	0.556	**0.051**	**0.051**	**0.049**	0.048	**0.050**	0.049
2.0	0.500	**0.050**	**0.050**	0.051	**0.050**	**0.050**	**0.050**
2.2	0.455	**0.051**	0.052	0.051	**0.050**	0.054	**0.051**
2.4	0.417	**0.051**	**0.051**	0.049	**0.051**	**0.053**	0.054
2.6	0.385	**0.050**	**0.050**	**0.053**	**0.053**	**0.051**	0.049
2.8	0.357	0.049	**0.050**	**0.050**	0.053	**0.047**	0.046
3.0	0.333	**0.052**	**0.052**	0.049	**0.050**	**0.052**	0.056
3.2	0.313	**0.051**	**0.051**	**0.050**	0.048	**0.049**	**0.049**
3.4	0.294	**0.051**	**0.051**	**0.053**	0.054	0.049	**0.050**
3.6	0.278	**0.050**	0.052	0.052	**0.049**	**0.049**	0.043
3.8	0.263	**0.050**	**0.050**	**0.048**	0.047	**0.050**	**0.050**
4.0	0.250	0.049	**0.050**	**0.051**	**0.051**	**0.050**	**0.050**
5.0	0.200	**0.049**	**0.049**	0.052	**0.051**	0.048	**0.050**
6.0	0.167	0.051	**0.050**	**0.051**	0.052	**0.050**	0.046
7.0	0.143	**0.051**	**0.051**	**0.050**	0.051	0.055	**0.051**
8.0	0.125	0.051	**0.050**	**0.050**	0.051	0.053	**0.050**
9.0	0.111	**0.050**	**0.050**	**0.051**	**0.051**	**0.051**	0.047
10.0	0.100	**0.050**	**0.050**	0.052	**0.051**	**0.053**	**0.053**

Table 3: Results of power of the ELR and ELR2 tests for the mixture of two exponentials

$\lambda_1=1$	$n=10$		$n=20$		$n=50$		$n=100$		$n=200$	
λ_2	ELR	ELR2	ELR	ELR2	ELR	ELR2	ELR	ELR2	ELR	ELR2
1.0	0.050	**0.051**	**0.050**	0.049	**0.052**	0.047	**0.053**	**0.053**	0.049	**0.050**
1.2	**0.053**	**0.053**	**0.050**	**0.050**	**0.056**	0.055	**0.055**	0.052	**0.057**	**0.057**
1.4	**0.057**	**0.057**	**0.063**	0.060	**0.068**	0.066	**0.073**	0.067	**0.085**	0.081
1.6	**0.064**	0.063	**0.069**	0.067	0.078	**0.079**	**0.094**	0.092	**0.119**	0.114
1.8	**0.070**	0.069	**0.079**	0.078	**0.109**	0.106	**0.128**	0.124	**0.182**	0.172
2.0	**0.079**	0.078	0.092	**0.095**	**0.131**	0.129	**0.179**	0.172	**0.259**	0.247
2.2	**0.089**	0.086	**0.109**	0.105	**0.156**	**0.156**	**0.238**	0.226	**0.355**	0.349
2.4	**0.099**	0.098	0.122	**0.126**	**0.202**	0.201	**0.300**	0.296	**0.464**	0.452
2.6	**0.109**	0.108	**0.142**	**0.142**	**0.244**	0.241	0.370	**0.371**	**0.568**	0.566
2.8	**0.122**	0.121	**0.163**	0.159	**0.280**	0.278	0.436	**0.439**	**0.670**	0.669
3.0	**0.131**	0.128	**0.182**	**0.182**	0.316	**0.328**	0.513	**0.516**	0.752	**0.759**
3.2	**0.144**	0.141	**0.210**	**0.210**	**0.371**	**0.371**	0.581	**0.590**	0.820	**0.829**
3.4	**0.159**	0.158	0.225	**0.231**	0.410	**0.423**	0.648	**0.663**	0.880	**0.888**
3.6	**0.170**	0.169	0.247	**0.259**	0.460	**0.479**	0.705	**0.718**	0.916	**0.927**
3.8	**0.182**	**0.182**	0.274	**0.279**	0.496	**0.516**	0.752	**0.766**	0.947	**0.956**
4.0	**0.197**	0.195	0.288	**0.305**	0.547	**0.573**	0.801	**0.825**	0.967	**0.975**

2.2 RT class tests for normality

The general RT class is based on robustification of the classical Jarque-Bera test introduced by [Bera and Jarque (1981)]. The general RT class test statistic was defined by [Stehlík et al. (2012)] for purpose of robust testing for normality against Pareto tails and has the following general form

$$RT = \frac{k_1(n)}{C_1}\left(\frac{M_{j_1}^{\alpha_1}(r_1, T_{(i_1)}(s_1))}{M_{j_2}^{\alpha_2}(r_2, T_{(i_2)}(s_2))} - K_1\right)^2 + \frac{k_2(n)}{C_2}\left(\frac{M_{j_3}^{\alpha_3}(r_3, T_{(i_3)}(s_3))}{M_{j_4}^{\alpha_4}(r_4, T_{(i_4)}(s_4))} - K_2\right)^2, \quad (5)$$

where M_j are jth theoretical central moment estimators of the random variable defined as $M_j(r, T(F_n, s)) = \frac{1}{n-2r}\sum_{m=r+1}^{n-r}\varphi_j(X_{(m)} - T(F_n, s))$ for $j \in \{0, 1, 2, 3, 4\}$, where φ_j is a tractable and continuous function, where $\varphi_0(x) = \sqrt{\pi/2}|x|$ and $\varphi_j(x) = x^j$ for $j \in \{1, 2, 3, 4\}$, $X_{(m)}$ is the order statistic, $T(F_n, s)$ is a location functional applied to the sample $X_1, X_2, \ldots, X_n$, r and s are the trimming constants for moments and location, respectively, K_1 and K_2 are small-sample variants of mean corrections, C_1 and C_2 are asymptotic constants, α_1, α_2, α_3 and α_4 are exponents, and finally, $k_1(n)$ and $k_2(n)$ are functions of sample size n. In the general RT class we used the following four different location estimators:

- mean: $T_{(0)} = \frac{1}{n}\sum_{i=1}^{n} X_i$,
- median: $T_{(1)} = F_n^{-1}(1/2)$,
- trimmed mean: $T_{(2)}(s) = \frac{1}{n-2s}\sum_{i=s+1}^{n-s} X_{(i)}$, where $X_{(i)}$ is the i-th order statistic of the sample and s is the trimming constant for location,
- pseudo-median: $T_{(3)} = \text{median}_{i\leq j}(X_i + X_j)/2$, i.e. the median of the set $\{(X_1 + X_1)/2, (X_1 + X_2)/2, (X_1 + X_3)/2, \ldots, (X_1 + X_n)/2, (X_2 + X_2)/2, (X_2 + X_3)/2, \ldots, (X_2 + X_n)/2, \ldots, (X_{n-1} + X_n)/2, (X_n + X_n)/2\}$.

Some detailed information about the general RT class test statistic as well as some theoretical results on consistency and asymptotic χ^2-distribution of this statistic and some geometric aspects of robust testing for normality can be found in [Stehlík et al. (2012)], [Stehlík et al. (2014)] and [Richter et al. (2017)].

By clustering based on power values from all analyzed tests of RT class the following representatives with good properties have been obtained (for more see [Stehlík et al. (2014)]):

- The mean-median $MMRT1$ test with test statistic:

$$MMRT1 = \frac{n}{18}\left(\frac{M_3(0, T_{(1)}(F_n, 0))}{M_2^{3/2}(0, T_{(0)}(F_n, 0))}\right)^2 + \frac{n}{24}\left(\frac{M_4(0, T_{(0)}(F_n, 0))}{M_2^2(0, T_{(0)}(F_n, 0))} - 3\right)^2.$$

- The mean-median $MMRT2$ test with test statistic:

$$MMRT2 = \frac{n}{18}\left(\frac{M_3(0, T_{(1)}(F_n, 0))}{M_2^{3/2}(0, T_{(1)}(F_n, 0))}\right)^2 + \frac{n}{24}\left(\frac{M_4(0, T_{(0)}(F_n, 0))}{M_2^2(0, T_{(1)}(F_n, 0))} - 3\right)^2.$$

- The mean-median $TTRT1$ test with trimming $s = r = 0.05n$ with test statistic:

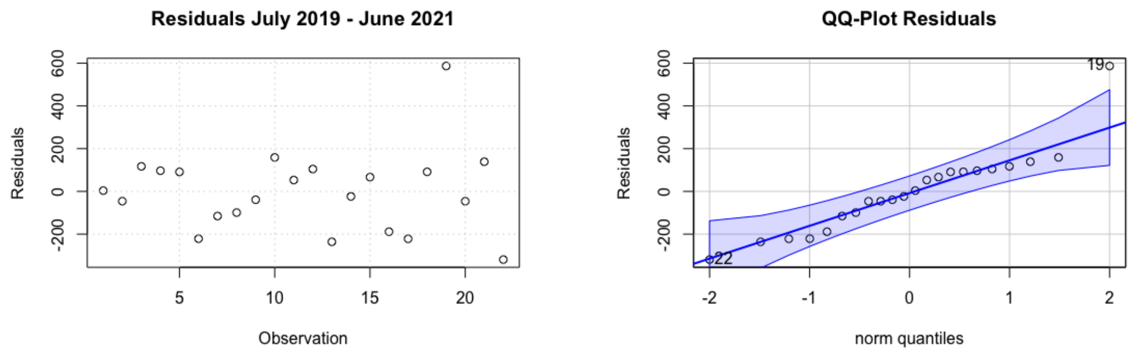
$$TTRT1 = \frac{4n}{5}\left(\frac{M_3(0.05n, T_{(2)}(F_n, 0.05n))}{M_2^{3/2}(0, T_{(0)}(F_n, 0))}\right)^2 + \frac{27n}{20}\left(\frac{M_4(0.05n, T_{(2)}(F_n, 0.05n))}{M_2^2(0, T_{(0)}(F_n, 0))} - 0.85\right)^2.$$

- The mean-median $TTRT2$ test with trimming $s = r = 0.05n$ with test statistic:

$$TTRT2 = \frac{16n}{5}\left(\frac{M_3(0.05n, T_{(2)}(F_n, 0.05n))}{M_2^{3/2}(0.05n, T_{(2)}(F_n, 0.05n))}\right)^2 + \frac{n}{550}\left(\frac{M_4(0, T_{(0)}(F_n, 0))}{M_2^2(0.05n, T_{(2)}(F_n, 0.05n))} - 7.73\right)^2.$$

3 IPSA modelling

The Index of Selective Stock Prices (IPSA for its acronym in Spanish) is the primary stock market performance indicator of the Chilean Stock Exchange that considers the leading traded stocks. Since 2018 it has been called S&P/CLX IPSA after the alliance between S&P Down Jones Indices and the Santiago Stock Exchange. These data were first applied in [Stehlík et al. (2023)], and here we test for the normal distribution of the residuals. The window time is from July 2019 to June 2021, see Figure 1.



(a) Residuals (b) QQ-Plot

Figure 1: Residuals and QQ-Plot

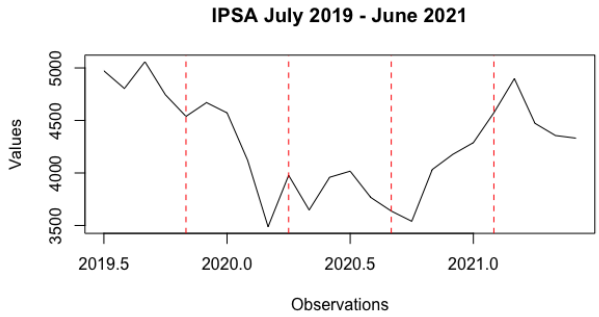
Table 4: Residual Normally tests

	statistic	p-value
Shapiro-Wilk	0.90258	0.03349
Kolmogorov-Smirnov	0.5	0.00001306
Anderson-Darling	0.58652	0.1133
Lilliefors	0.15433	0.1885

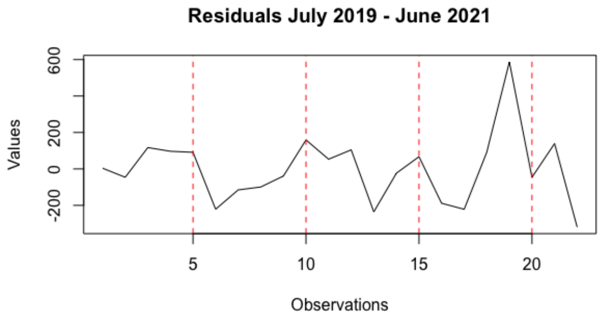
As we can see in Table 4, the results are divided, and the most different result is the Kolmogorov-Smirnov test. However, this case is vastly studied, and it is not recommended in some instances where we work with a small sample, and when the parameters are estimates from real data, you can see, e.g., [Ghasemi A, Zahediasl S. (2012)],[Steinskog et al. (2007)], [Marsaglia et al. (2003)] and references therein. Continuing with the residual analysis, we apply the methodology presented for [Orlando et al. (2019)] that divides the dataset into sub-set of data. The algorithm starts with a minimum of observation, in our case, the first five, and then the Lilliefords test is run; if the result is normal, then the algorithm continue with the following five observations. If the Lilliefors test is not normal, the algorithm adds new observations until the test does not rejects normality. This is repeated several times with the entire dataset. We also applied JB, LT, RJB, SW, MMRT1, MMRT2, TTRT1, TTRT2 tests developed in [Stehlík et al. (2014)] with satisfactory results of a robust normality.

Table 5: Results of testing for normality for the analyzed data sets

Data	IPSA		Subset 1		Subset 2		Subset 3		Subset 4	
Test	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
JB	1.144	0.379	0.362	0.783	0.267	0.883	0.579	0.491	1.132	0.094
LT	0.088	0.904	0.154	0.943	0.169	0.873	0.231	0.414	0.217	0.513
RJB	0.868	0.468	0.221	0.844	0.200	0.868	0.457	0.468	1.298	0.153
SW	0.965	0.551	0.975	0.917	0.959	0.813	0.933	0.610	0.859	0.184
MMRT1	1.237	0.390	0.439	0.714	0.156	0.938	0.648	0.523	1.546	0.134
MMRT2	1.278	0.386	0.474	0.696	0.157	0.942	0.683	0.510	1.234	0.178
TTRT1	4.214	0.407	4.211	0.713	10.437	0.389	1.837	0.927	25.643	0.118
TTRT2	1.128	0.497	0.402	0.897	0.593	0.580	0.436	0.822	2.370	0.093



(a) Raw IPSA



(b) Residuals

Figure 2: Subsets Lilliefors Test

Analyzing a subset of the time series is essential to find clusters of volatility or jumps caused by some announcement of an influential person like the study in [Stehlík et al. (2023)] or decisions by central banks like in the case of the Monetary Policy Rate. In our case (see Figure 2), for both the original dataset of IPSA and the residuals from [Stehlík et al. (2023)], the subset was compounded by five observations showing normality by the Lilliefors test. Tables 5 and 6 present the p-values for each subset for both data sets.

Table 6: IPSA and Residuals p-values Lilliefors test

Subsets	IPSA p-values	Residuals p-values
1	0.9154639	0.1217613
2	0.7210761	0.5064873
3	0.6182405	0.2654715
4	0.6839121	0.4302534

Following the same idea presented above, we use the same algorithm but, at this time, we apply an exponential test; as a result, specifically the Moran test. The reason to use the Moran test is the fact it is pivotal; see [Moran (1951)], [Stehlík (2003)], [Stehlík (2006)]. Table 7 represents the p-values for raw data for IPSA in June 2019 and July 2021.

Table 7: IPSA p-values Moran test

Subsets	IPSA p-values
1	0.984
2	0.985
3	0.986
4	0.988
5	0.9845
6	0.987

Considering the Moran‘s test, the subsets were compounded by four observations, which we see in the following Figure 3.

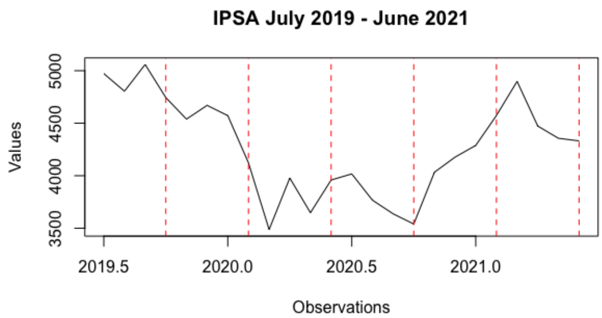
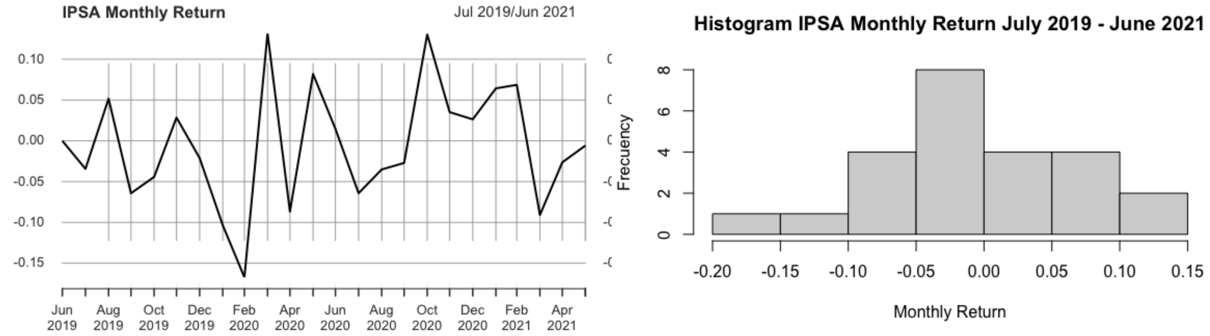


Figure 3: Subsets Moran test

3.1 Monthly Return Analysis of IPSA

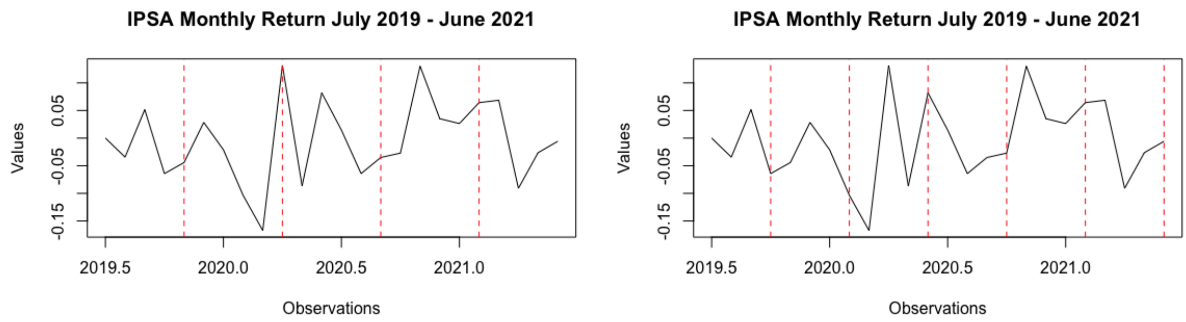
A traditional calculation for working with stock prices or indexes is the logarithm of returns. We can compare different assets based on their profitability and not their price. Below we present the monthly IPSA's return in the analysis period and a histogram that reflects the behavior of its returns, see Figure 4. We checked several classical and robust normality tests (JB, LT, RJB, SW, MMRT1, MMRT2, TTRT1 and TTRT2) for all monthly returns, and also for subsets 1-4 (each having $n = 5$ observations) and null hypothesis of normality was not rejected. For monthly returns subsets see Figure 5.

Following the analysis above, we apply the same algorithm with Lilliefors and Moran tests to detect subsets, but this time with the monthly return of IPSA to understand the volatility clusters from an economic point of view. The results of subsets are the same as for raw IPSA.



(a) Monthly Return (b) Histogram of the Returns

Figure 4: Monthly Returns of IPSA



(a) Monthly Return Lilliefors Test (b) Monthly Return Moran Test

Figure 5: Monthly Returns Subsets

Table 8: Results of testing for normality for monthly returns

Data	Monthly returns		Subset 1		Subset 2		Subset 3		Subset 4	
Test	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
JB	0.087	0.959	0.537	0.555	0.527	0.568	0.749	0.272	0.449	0.676
LT	0.083	0.938	0.203	0.620	0.196	0.670	0.199	0.653	0.166	0.891
RJB	0.069	0.962	0.412	0.555	0.364	0.641	0.689	0.243	0.344	0.673
SW	0.985	0.962	0.951	0.754	0.945	0.704	0.917	0.486	0.935	0.627
MMRT1	0.232	0.873	0.660	0.512	0.816	0.391	0.868	0.358	0.383	0.764
MMRT2	0.267	0.856	0.706	0.491	0.892	0.351	0.813	0.407	0.391	0.769
TTRT1	2.681	0.558	2.888	0.838	3.523	0.779	19.138	0.190	8.552	0.456
TTRT2	0.882	0.619	0.502	0.684	0.551	0.622	1.619	0.178	0.796	0.434

In table 9 we present the parameters for normal and exponential distributions for all subsets; for the normal case, we used regular mean and standard deviation, and for the exponential (a) case, we transform the data linearly with the form $\hat{y} = x + a$ taken arbitrary selection of $a = 1$. We calculate $\hat{\lambda}$ by MLE for this case in each group, for values see Table 9, column 4.

In the exponential (b) case, we use the two-parameter exponential distribution

$$\frac{1}{\sigma}\exp(-\frac{(x-\mu)}{\sigma}). \quad (6)$$

For this case 6 we calculated the parameter lambda as follows: $\hat{\lambda}_i = \frac{n-1}{\bar{x}-x_{imin}}$. This is based on results of [Epstein (1956)], where complete and sufficient statistics for (μ, σ) of is derived as $(x_{(1)}, V)$, $V = \frac{n}{n-1}(\bar{x} - x_{(1)})$, $\bar{x}$ is arithmetic mean and $x_{(1)} := min(x)$. The reason for using two-exponential model (6) is that we do not know lower range of values for each group, and since the minimum varies in each group, we shall estimate it. We can see substantially higher variation of the estimated values $\hat{\lambda}_i$, see Table 9, column 5.

Table 9: Parameters of the Subsets

Distribution	Normal		Exponential		Exponential (a)	Exponential (b)
Parameter	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\lambda}$
Subset 1	-0.01824638	0.04547658	-0,01174663	0,049757887	1.011886	57.11886722
Subset 2	-0.02639882	0.1159322	-0.03510419	0.054751262	1.036381	43.87993878
Subset 3	-0.01788538	0.06741795	-0.01012370	0.140335861	1.010227	19.08575026
Subset 4	0.04580174	0.05767669	-0.02797310	0.03253285	1.028778	82.72798851
Subset 5	NA	NA	0.06403655	0.04710354	0.9398173	79,55341365
Subset 6	NA	NA	-0.01359393	0.06568482	1.013781	38,84608733

When we use model of parameter dependence (2) for column 4 (Exponential (a)) in Table 9, namely $\lambda_i = \lambda(\mu_i, \sigma_i^2)$ by simple ordinaly least squares regression we obtained

$$\hat{\lambda} = 1.0020160 - 0.0118259\hat{\sigma} - 0.9676346\hat{\mu}$$

with Multiple R-squared: 0.9996 and Adjusted R-squared: 0.9993, and for the column 5 (Exponential (b)), we received

$$\hat{\lambda} = 88.89 - 523.45\hat{\sigma} + 228.43\hat{\mu}$$

with Multiple R-squared: 0.8123 and Adjusted R-squared: 0.6872.

3.1.1 Setup for simulation

1. We run $M = 100{,}000$ simulations $X \sim exponential(\lambda = 1)$ for n equals the sample size $n = 24$.

2. Running simulation setups for the ELR, ELR2 a ELR3 tests to receive discretization for later computing of p-values.

3. For IPSA dataset we compute the ELR, ELR2 and ELR3 test statistics.

4. Computing of p-values.

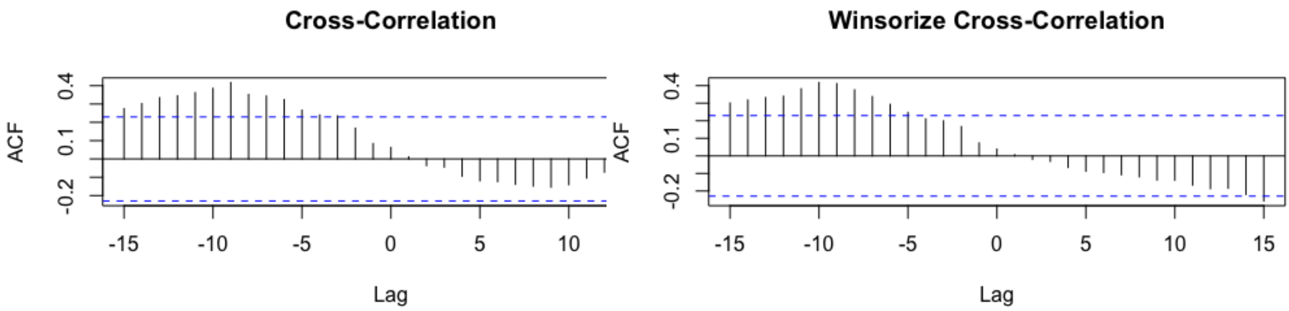
Table 10: Test statistics and p-values of the ELR, ELR2 and ELR3 tests for IPSA data

	statistic	p-value
ELR	0.1392	1.000
ELR2	0.1020	1.000
ELR3	0.1239	1.000

As we can see in Table 10, computed p-values are numerically equal 1.

3.1.2 IPSA and Interest Rate

The IPSA represents the return of the market comparable to other stock market indicators in different stock exchanges around the world, e.g., Ibex 35 in Spain, S&P 500 in the United States, and Bovespa for Brazil, among others. On the other hand, the interest rate represents the return obtained by a financial institution when lending money, and from the consumer's point of view, it is the cost of being able to use the money. The relationship between interest rates and the market return has been studied in different countries, and the result depends on the country analyzed; you can see ([Canova, Fabio, & Nicolo, d. (1997)], [Lee (1992)], [Gjerde, Ø. and Sættem, F. (1999)] and references therein). For our case, we realize a cross-correlation analysis between current interest rate and IPSA from 2016 to 2022.



(a) Cross-Correlation (CC) (b) Winzorize Cross-Correlation (WCC)

Figure 6: Cross-Correlation IPSA and Interes Rate

Figure 6 represents the correlation between IPSA and Interest rates at different lags, and Table 11 shows the cross-correlations values. We also calculate a robust estimation using R-package datawizard described in [Patil et al. (2022)].

Table 11: Auto-correlations of series by lag

Lag	CC	WCC	Lag	CC	WCC
0	0.063	0.038	0	0.063	0.038
-1	0.084	0.074	1	0.011	0.006
-2	0.168	0.167	2	-0.036	-0.020
-3	0.235	0.199	3	-0.045	-0.031
-4	0.240	0.210	4	-0.094	-0.066
-5	0.266	0.247	5	-0.119	-0.087
-6	0.324	0.293	6	-0.124	-0.095
-7	0.343	0.338	7	-0.138	-0.108
-8	0.352	0.376	8	-0.148	-0.119
-9	0.417	0.412	9	-0.154	-0.137
-10	0.385	0.417	10	-0.141	-0.138
-11	0.362	0.382	11	-0.104	-0.166
-12	0.344	0.339	12	-0.072	-0.187
-13	0.334	0.332	13	-0.079	-0.184
-14	0.303	0.318	14	-0.098	-0.220
-15	0.275	0.301	15	-0.143	-0.257

3.2 Formal check of non-zero cross-correlations

Based on high variability we decided to use several correlation measures, namely Pearson, Winsorized, Jackknifed, robust and Spearman. Thus, we can check for all these established correlation measures non-zero cross-correlations. Therefore we used formal test statistics for standard hypothesis of zero cross-correlation

$$H_0 : K_{XY} = 0 \text{ versus } K_{XY} \neq 0 \tag{7}$$

The test statistics obtained by [Rice and Shum (2019)] has the form

$$S_N = N \sum_{i=0}^{p} C(p)^2 \tag{8}$$

where $C(p)$ is selected cross-covariance from our portfolio of Pearson, Jackknifed, Spearman, etc and N is the sample size.

4 Conclusions and discussion

The obtained results are two-fold. We received simulation and modelling results regarding the new testing model by construction of empirical subgroups and parameter dependence models. Regarding the application of the new testing model on IPSA data the results are quite interesting. We could see that the interest rate and returns at the stock exchange market are positively correlated at least in several lags, i.e., higher interest rates go hand in hand to higher returns at the stock market. This is contrary to conventional knowledge. In finance it is taught that the correlation should go the other way around: higher rates would put a pressure on stock market prices. This is because if interest rates are on the rise, investment in stocks become relatively unattractive since investors would then prefer to invest in interest bearing bonds or other debt securities, not in stocks. Figure

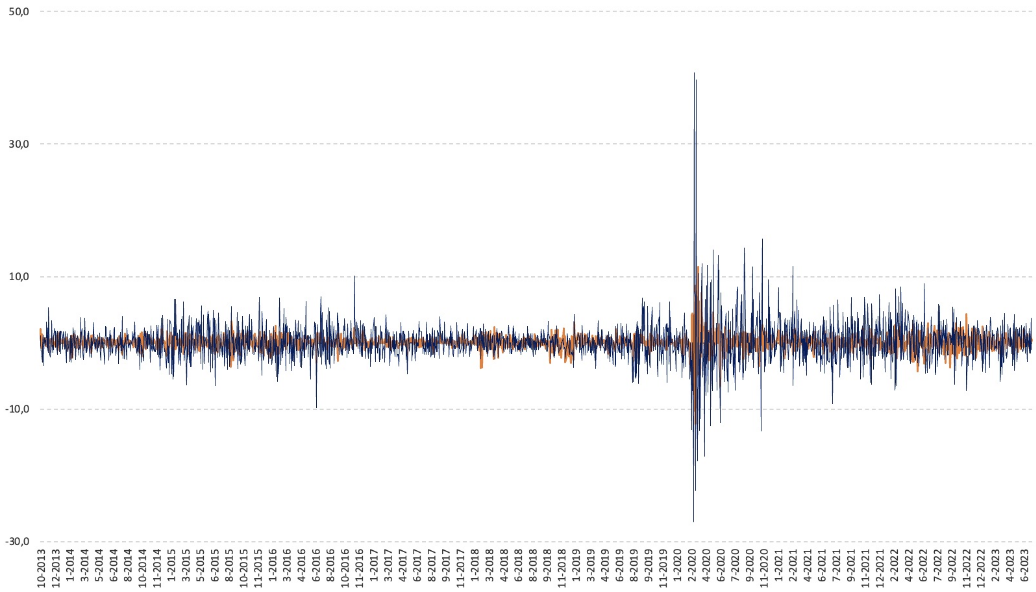


Figure 7: Dow Jones and long-term interest rates (US). Blue: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity (percent change). Red: Dow Jones Composite Average (percent change).

7 clearly depicts the long-term positive correlation between interest rates and stock market prices. Data provided by Board of Governors; S&P DJI, both retrieved from fred.stlouisfed.org.

We found a positive correlation between IPSA (stock market) returns and interest rate, and we concluded that this contrary to conventional wisdom in finance. However, we can intuitively understand why this conventional understanding can, at its best, only be true in the very short run. In the medium and long run, higher interest rates lead to faster growing deposits. All deposits taken together integrate the economy's money supply basically consisting of cash plus deposits. There exist different definitions of "money supply". The most commonly one used is the so-called broad money "M3" consisting of cash (bills and coins) and demand deposits (checking accounts) + all time-related deposits, saving deposits, non-institutional money-market funds + large and long-term deposits (2 years), institutional money-market funds, repurchase agreements, along with other larger liquid assets.

The faster money supply grows the more money will ceteris paribus be invested in investment markets such as stock or real estate. Also, to be able to pay the interest on faster growing deposits, banks have to grant more loans. Credits expansion then again increases the money supply via the so-called "money-multiplier" effect (see, e.g., [Mankiw and Taylor (2014)]). This in turn creates a potential for asset price inflation, i.e., an increase in stock market prices. This is why prices on the investment markets and the interest rates in the long run develop synchronously (see already [Fuders, Mondaca and Haruna (2013)], [Fuders and Max-Neef (2014)] and [Fuders (2021)]). We could prove here this heterodox interpretation and show that interest rates and stock market returns are indeed positively correlated.

5 Appendix

5.1 Kiguradze class interest rate

The general form of 2nd order interest rate model which naturally includes as the special cases standard interest rate models like CIR or Vašíček. By general form of 2nd order interest rate model we understood as arbitrary solution of (two dimensional) system of stochastic differential equation

$$\mathrm{d}\mathbf{X}_t = \tilde{\mathbf{b}}(t, \mathbf{X}_t)\mathrm{d}t + \mathbf{\Sigma}(t, \mathbf{X}_t)\mathrm{d}\mathbf{W}_t, \tag{9}$$

For $t \in I = [0, \infty)$ where $\mathbf{X}_t = (r_t, p_t)^\intercal$, r_t is interest rate and p_t represents generalized force of interest, and

$$\tilde{\mathbf{b}}(t, \mathbf{X}_t) = \left(c(t)(p_t)^m, a(t)(p_t)^l + b(t)(r_t)^n\right)^T, \mathbf{\Sigma}(t, \mathbf{X}_t) = \left(\begin{smallmatrix} 0 & 0 \\ 0 & \sigma(t)(r_t)^k \end{smallmatrix}\right)$$

where $a, b, c \in C(I)$, $c \neq 0$, $\sigma \geq 0$ and $n, k, l \in \mathbb{Q}$ and $m \in \mathbb{Q} \setminus \{0\}$ with odd denominators.

Trajectories for the specific values $m = 1, 3$ and development of United Kingdom interest rate from October 2019 to August 2020 has been studied in [Stehlík, et al. (2020)].

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Appendix C

CMF response

Santiago, 13 de enero de 2023

Estimado Sr. Leal,

Por medio de la presente, notifico a usted la decisión del Comité Editorial de la Comisión para el Mercado Financiero (CMF) de no considerar su propuesta de proyecto de investigación colaborativa en la Convocatoria de diciembre de 2022. El proyecto se titula "Statistical Modeling of the relationships between pension funds, inflation and interest rate during COVID-19 period in Chile," y la propuesta y sus términos de referencias fueron remitidos el 9 de diciembre pasado a la Comisión.

El Comité Editorial no cuestiona la relevancia del tema, pero, basado en los términos de referencia del proyecto, prioridades institucionales y recursos disponibles, estima que no se justifica la colaboración de la CMF en la investigación. En particular, no es claro que el que el trabajo requiera información de la CMF que no sea pública.

Agradecemos su interés por participar en esta convocatoria.

Atentamente,

DIRECTORA GENERAL ESTUDIOS, ESTADISTICAS Y DATOS
COMISION PARA EL MERCADO FINANCIERO

Appendix D

Codes

IPSA-Returns

```
rm(list = ls())
rm()



#Lista de los paquetes a utilizar
packages <- c("foreign","apaTables","PerformanceAnalytics","psych","corrr","corrplot","pipeR",
              "dbplyr","ggplot2","factoextra","scales","cluster","NbClust","tidyverse","tidyr",
              "tidyverse","purrr","dplyr","FactoMineR","readr","dbscan","quantmod","dtw","fpc",
              "readxl","nortest")



#Funci n ipak para instalar y llamar a varios paquetes a la vez

ipak <- function(pkg){
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}

#Llamar los paquetes del listado anterior
ipak(packages)

#Datos IPSA
data <- read_excel("Ipsa2016-2022-2.xlsx")
dataNumeric <- as.numeric(unlist(data[,2]))
ts_data <- ts(data[,2], frequency = 12, start = c(2016,1))
ts.plot(ts_data)

###Retorno Mensual 2016 - 2022#####

MR <- monthlyReturn(ts_data, type = "log")
head(MR)
plot(MR)
hist(x = MR, main = "Monthly Return 2016 - 2022", xlab = "Monthly Return", ylab = "Frecuency")
```

```
#Datos IPSA (July2019 - June2021)
ts_data719_621 <- ts(data[43:66,2], frequency = 12, start = c(2019,7)
    )
head(ts_data719_621)
ts.plot(ts_data719_621)


###Retorno Mensual July 2019 - June 2021#####

MR719_621 <- monthlyReturn(ts_data719_621, type = "log")
head(MR719_621)
plot(x = MR719_621, main = "IPSA Monthly Return")
hist(x = MR719_621, main = "Histogram IPSA Monthly Return July 2019 -
     June 2021", xlab = "Monthly Return", ylab = "Frecuency")

### 4 Subsets of normal distribution (group of 5 observations)###

Subset1 <- ts(MR719_621[0:5], frequency = 12, start = c(2019,7))
head(Subset1)
mean(Subset1)
sd(Subset1)
min(Subset1)

Subset2 <- ts(MR719_621[6:10], frequency = 12, start = c(2019,12))
head(Subset2)
mean(Subset2)
sd(Subset2)
min(Subset2)

Subset3 <- ts(MR719_621[11:15], frequency = 12, start = c(2020,5))
head(Subset3)
mean(Subset3)
sd(Subset3)
min(Subset3)

Subset4 <- ts(MR719_621[16:20], frequency = 12, start = c(2020,10))
head(Subset4)
mean(Subset4)
sd(Subset4)
min(Subset4)

### 6 Subsets of exponential distribution (group of 4 observations)
    ###
```

```
expSubset1 <- ts(MR719_621[0:4], frequency = 12, start = c(2019,7))
head(expSubset1)
expSubset1 <- expSubset1 + 1
expSubset1
min(expSubset1)
lambda1 <- 1/mean(expSubset1)
lambda1

expSubset2 <- ts(MR719_621[5:8], frequency = 12, start = c(2019,12))
head(expSubset2)
expSubset2 <- expSubset2 + 1
expSubset2
min(expSubset2)
lambda2 <- 1/mean(expSubset2)
lambda2

expSubset3 <- ts(MR719_621[9:12], frequency = 12, start = c(2020,5))
head(expSubset3)
expSubset3 <- expSubset3 + 1
expSubset3
min(expSubset3)
lambda3 <- 1/mean(expSubset3)
lambda3

expSubset4 <- ts(MR719_621[13:16], frequency = 12, start = c(2020,10))
head(expSubset4)
expSubset4 <- expSubset4 + 1
expSubset4
min(expSubset4)
lambda4 <- 1/mean(expSubset4)
lambda4

expSubset5 <- ts(MR719_621[17:20], frequency = 12, start = c(2020,10))
head(expSubset5)
expSubset5 <- expSubset5 + 1
expSubset5
min(expSubset5)
lambda5 <- 1/mean(expSubset5)
lambda5
```

```
expSubset6 <- ts(MR719_621[21:24], frequency = 12, start = c(2020,10))
head(expSubset6)
expSubset6 <- expSubset6 + 1
expSubset6
min(expSubset6)
lambda6 <- 1/mean(expSubset6)
lambda6

lambda1
lambda2
lambda3
lambda4
lambda5
lambda6
```

OLSExponentialLambda

```
    la <- c(1.011886, 1.036381, 1.010227, 1.028778, 0.9398173, 1.013781)
plot(la)

lb <- c(57.11886722,43.87993878, 19.08575026, 82.72798851, 79.55341365, 38.84608733 )
plot(lb)

s <- c(0.049757887, 0.054751262, 0.140335861, 0.03253285, 0.04710354, 0.06568482)
plot(s)

m <- c(-0.01174663, -0.03510419, -0.01012370, -0.02797310, 0.06403655, -0.01359393)
plot(m)

za<-lm(la~s+m)
summary(za)

zb<-lm(lb~s+m)
summary(zb)
```

ReturnIpsaLillieforsMoran

```
    rm(list = ls())
rm()


```

```
#Lista de los paquetes a utilizar
packages <- c("foreign","apaTables","PerformanceAnalytics","psych","corrr","corrplot","pipeR",
              "dbplyr","ggplot2","factoextra","scales","cluster","NbClust","tidyverse","tidyr",
              "tidyverse","purrr","dplyr","FactoMineR","readr","dbscan","quantmod","dtw","fpc",
              "readxl","nortest","exptest")



#Funci n ipak para instalar y llamar a varios paquetes a la vez
ipak <- function(pkg){
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}

#Llamar los paquetes del listado anterior
ipak(packages)

#########
# DATOS #
#########

#Datos IPSA
data <- read_excel("Ipsa2016-2022-2.xlsx")
#dataNumeric <- as.numeric(unlist(data[,2]))
ts_data <- ts(data[,2], frequency = 12, start = c(2016,1))
ts_data <- ts(monthlyReturn(ts_data, type = "log"), frequency = 12, start = c(2016,1))
head(ts_data)
ts.plot(ts_data)          #Plot 1
hist(x = ts_data, main = "Monthly Return 2016 - 2022", xlab = "Monthly Return", ylab = "Frecuency")

#Datos July 2019 - June2021
ts_data619_721 <- ts(data[43:66,2], frequency = 12, start = c(2016,1))
ts_data619_721 <- ts(monthlyReturn(ts_data619_721, type = "log"), frequency = 12, start = c(2019,7))


```

```
#Datos u
# u <- c(3.707753, -45.970849, 117.088626, 96.778142, 91.026379,
   -221.263028, -115.287173, -99.248793,
#         -39.029997, 158.888870, 53.167747, 104.576030, -235.949681,
   -23.587276, 66.779914, -188.467175,
#         -221.742604, 91.513380, 586.679662, -46.190226, 138.867814,
   -319.017637)
# ts_u <- ts(as.numeric(unlist(u)))

###################
# TEST LILLIEFORS #
###################
kol_smir <- function(datos,h){
    n = list()
    p = list()
    a = 1
    b = h
    #h = 5 #Lilliefors no admite 4 datos
    while(TRUE){
        lillie_test <- lillie.test(datos[a:b])

        #Si el valor p es mayor a 0.05
        if(lillie_test$p.value > 0.05){
            n <- append(n,b)
            a = b + 1
            b = b + h
            p <- append(p,lillie_test$p.value)

        }
        else{
            b = b + 1
        }

        #Si se acaban los datos
        if(b > length(datos)){
            l <- list("n" = n,"p" = p)
            return(l)
        }
    }
}

#################
# TEST DE MORAN #
#################
```

```
moran <- function(datos,h){
  n = list()
  p = list()
  a = 1
  b = h
  while(TRUE){
    moran_test <- moran.exp.test(datos[a:b], simulate.p.value=TRUE,
   nrepl=2000)

    #Si el valor p es mayor a 0.05
    if(moran_test$p.value > 0.05){
      n <- append(n,b)
      a = b + 1
      b = b + h
      p <- append(p,moran_test$p.value)
    }
    else{
      b = b + 1
    }

    #Si se acaban los datos
    if(b > length(datos)){
      l <- list("n" = n,"p" = p)
      return(l)
    }
  }
}

########
# PLOT #
########
plot_grafico <- function(datos,cortes,title,sub){
  #Si hay algo de la configuraci?n del plotpara cada variable basta
   comentar la l?nea (#) y
  #compilar nuevamente la funci?n
  plot(datos,
       plot.type = "single",
       main = title,
       xlab="Observations",
       ylab="Values")

  times_ts = time(datos)

  for(corte in cortes){
```

```r
    abline(v = times_ts[corte], col = "red",lwd=1, lty=2)
  }
}

###############
# LILLIE TEST #
###############
values <- kol_smir(ts_data,5)

cortesLillie <- values$n
print(cortesLillie)

p_valuesIPSA <- values$p
print(p_valuesIPSA)

plot_grafico(ts_data,cortesLillie,"IPSA (LILLIE)","2016-2022")

#Datos IPSA (Lillie) (July2019 - June2021)

values <- kol_smir(ts_data619_721,5)

cortes619_721 <- values$n
print(cortes619_721)

p_values619_721 <- values$p
print(p_values619_721)

plot_grafico(ts_data619_721,cortes619_721,"IPSA Monthly Return July 2019 - June 2021")

#####
# u #
#####
# values <- kol_smir(ts_u,5)
#
# cortesU <- values$n
# print(cortesU)
#
# p_valuesU <- values$p
# print(p_valuesU)
#
# plot_grafico(ts_u,cortesU,"Residuals July 2019 - June 2021")

################
```

```
# TEST DE MORAN #
#################
values <- moran(ts_data,4)

cortesMoran <- values$n
print(cortesMoran)

p_valuesMoran <- values$p
print(p_valuesMoran)

plot_grafico(ts_data,cortesMoran,"IPSA (Moran) 2016-2022")

#Datos IPSA (Moran) (July2019 - June2021)
values <- moran(ts_data619_721,4)

cortesIPSA <- values$n
print(cortesIPSA)

p_valuesIPSA <- values$p
print(p_valuesIPSA)

plot_grafico(ts_data619_721,cortesIPSA,"IPSA Monthly Return July 2019 - June 2021")
```

Residuos

```
    library("car")
library("nortest")

u_1 <- c(0, 3.707753, -45.970849, 117.088626, 96.778142, 91.026379, -221.263028, -115.287173, -99.248793, -39.029997, 158.888870, 53.167747)
u_2 <-  c(0, 104.576030, -235.949681, -23.587276, 66.779914, -188.467175, -221.742604, 91.513380, 586.679662, -46.190226, 138.867814, -319.017637)
u <- c(3.707753, -45.970849, 117.088626, 96.778142, 91.026379, -221.263028, -115.287173, -99.248793, -39.029997, 158.888870, 53.167747, 104.576030, -235.949681, -23.587276, 66.779914, -188.467175, -221.742604, 91.513380, 586.679662, -46.190226, 138.867814, -319.017637)

# mean(u_1)
# mean(u_2)
mean(u)

```

```
# plot(u_1)
# plot(u_2)
plot(u, main = "Residuals July 2019 - June 2021", xlab = "Observation
    ", ylab = "Residuals" )
grid()

# qqPlot(u_1)
# qqPlot(u_2)
qqPlot(u, main = "QQ-Plot Residuals", ylab = "Residuals")

# shapiro.test(u_1)
# shapiro.test(u_2)
shapiro.test(u)

# ks.test(u_1, 'pnorm')
# ks.test(u_2, 'pnorm')
ks.test(u, 'pnorm')

# ad.test(u_1)
# ad.test(u_2)
ad.test(u)

# lillie.test(u_1)
# lillie.test(u_2)
lillie.test(u)
```

LillieforsMoran

```
    rm(list = ls())
rm()


#Lista de los paquetes a utilizar
packages <- c("foreign","apaTables","PerformanceAnalytics","psych","
    corrr","corrplot","pipeR",
              "dbplyr","ggplot2","factoextra","scales","cluster","
    NbClust","tidyverse","tidyr",
              "tidyverse","purrr","dplyr","FactoMineR","readr","
    dbscan","quantmod","dtw","fpc",
              "readxl","nortest","exptest")



#Funci n ipak para instalar y llamar a varios paquetes a la vez
ipak <- function(pkg){
```

```
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}

#Llamar los paquetes del listado anterior
ipak(packages)

#########
# DATOS #
#########

#Datos IPSA
data <- read_excel("Ipsa2016-2022-2.xlsx")
#dataNumeric <- as.numeric(unlist(data[,2]))
ts_data <- ts(data[,2], frequency = 12, start = c(2016,1))
ts.plot(ts_data)

#Datos u
u <- c(3.707753, -45.970849, 117.088626, 96.778142, 91.026379,
    -221.263028, -115.287173, -99.248793,
        -39.029997, 158.888870, 53.167747, 104.576030, -235.949681,
    -23.587276, 66.779914, -188.467175,
        -221.742604, 91.513380, 586.679662, -46.190226, 138.867814,
    -319.017637)
ts_u <- ts(as.numeric(unlist(u)))

###################
# TEST LILLIEFORS #
###################
kol_smir <- function(datos,h){
    n = list()
    p = list()
    a = 1
    b = h
    #h = 5 #Lilliefors no admite 4 datos
    while(TRUE){
        lillie_test <- lillie.test(datos[a:b])

        #Si el valor p es mayor a 0.05
        if(lillie_test$p.value > 0.05){
            n <- append(n,b)
            a = b + 1
```

```
            b = b + h
            p <- append(p,lillie_test$p.value)

        }
        else{
            b = b + 1
        }

        #Si se acaban los datos
        if(b > length(datos)){
            l <- list("n" = n,"p" = p)
            return(l)
        }
    }
}

################
# TEST DE MORAN #
################
moran <- function(datos,h){
  n = list()
  p = list()
  a = 1
  b = h
  while(TRUE){
    moran_test <- moran.exp.test(datos[a:b], simulate.p.value=TRUE,
   nrepl=2000)

    #Si el valor p es mayor a 0.05
    if(moran_test$p.value > 0.05){
      n <- append(n,b)
      a = b + 1
      b = b + h
      p <- append(p,moran_test$p.value)
    }
    else{
      b = b + 1
    }

    #Si se acaban los datos
    if(b > length(datos)){
      l <- list("n" = n,"p" = p)
      return(l)
    }
```

```
  }
}

########
# PLOT #
########
plot_grafico <- function(datos,cortes,title,sub){
  #Si hay algo de la configuraci?n del plotpara cada variable basta
    comentar la l?nea (#) y
  #compilar nuevamente la funci?n
  plot(datos,
       plot.type = "single",
       main = title,
       xlab="Observations",
       ylab="Values")

  times_ts = time(datos)

  for(corte in cortes){
    abline(v = times_ts[corte], col = "red",lwd=1, lty=2)
  }
}

########
# IPSA #
########
values <- kol_smir(ts_data,5)

cortesIPSA <- values$n
print(cortesIPSA)

p_valuesIPSA <- values$p
print(p_valuesIPSA)

plot_grafico(ts_data,cortesIPSA,"IPSA","2016-2022")

#Datos IPSA (July2019 - June2021)
ts_data619_721 <- ts(data[43:66,2], frequency = 12, start = c(2019,7)
   )

values <- kol_smir(ts_data619_721,5)

cortes619_721 <- values$n
print(cortes619_721)
```

```
p_values619_721 <- values$p
print(p_values619_721)

plot_grafico(ts_data619_721,cortes619_721,"IPSA July 2019 - June 2021
    ")

#####
# u #
#####
values <- kol_smir(ts_u,5)

cortesU <- values$n
print(cortesU)

p_valuesU <- values$p
print(p_valuesU)

plot_grafico(ts_u,cortesU,"Residuals July 2019 - June 2021")

#################
# TEST DE MORAN #
#################
values <- moran(ts_data,4)

cortesIPSA <- values$n
print(cortesIPSA)

p_valuesIPSA <- values$p
print(p_valuesIPSA)

plot_grafico(ts_data,cortesIPSA,"IPSA 2016-2022")

#Datos IPSA (July2019 - June2021)
values <- moran(ts_data619_721,4)

cortesIPSA <- values$n
print(cortesIPSA)

p_valuesIPSA <- values$p
print(p_valuesIPSA)

plot_grafico(ts_data619_721,cortesIPSA,"IPSA July 2019 - June 2021")
```

Inflation

```
    library(readxl)
library(tidyverse)
library(lubridate)



data <- read.csv("serie-historica-empalmada-ipc-diciembre-2009-a-la-
    fecha.csv", sep = ";")

##View(data)
Inflation <- ts(data[,4], start = c(2009, 12), frequency = 12)
Inflation
plot(Inflation, main = "Monthly variation of the consumer price index
     (IPC in Spanish)")

Inflation2 <- ts(data[,4], start = c(2019, 1), end = c(2021, 10),
    frequency = 12)
Inflation2
plot(Inflation2, main = "Monthly variation of the consumer price
    index (IPC in Spanish)")
grid()

IPC <- read_xlsx("IPC20192022.xlsx")
IPC
IPC20192022 <- ts(IPC[,2], start = 2019, frequency = 12)
IPC20192022
plot(IPC20192022, main = "Monthly variation of the consumer price
    index (2019-2022)")
grid()
```

GDPPerCapita

```
    library(readxl)
library(tidyverse)
library(lubridate)
library(readr)

data <- read_delim("GDP.csv", ";", escape_double = FALSE,
                   trim_ws = TRUE)

GDP <- ts(data [,2], start = 1960, frequency = 1)
GDP
plot(GDP, main = "GDP Per Capita 1960 - 2020")

```

```
GDP2 <- ts(data [41:61,2], start = 2000, frequency = 1)
GDP2
plot(GDP2, main = "GDP Per Capita 2000 - 2020")

GDP3 <- ts(data [58:61,2], start = 2017, frequency = 1)
GDP3
plot(GDP3, main = "GDP Per Capita 2017 - 2020")
```

Regression

```
    rm(list = ls())
rm()



#Lista de los paquetes a utilizar
packages <- c("foreign","apaTables","PerformanceAnalytics","psych","
    corrr","corrplot","pipeR",
              "dbplyr","ggplot2","factoextra","scales","cluster","
    NbClust","tidyverse","tidyr",
              "tidyverse","purrr","dplyr","FactoMineR","readr","
    dbscan","quantmod","dtw","fpc",
              "readxl","nortest", "car")



#Funci n ipak para instalar y llamar a varios paquetes a la vez

ipak <- function(pkg){
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}

#Llamar los paquetes del listado anterior
ipak(packages)

#Datos
data <- read_excel("Regresion.xlsx")
dataNumeric <- as.numeric(unlist(data[,2]))
m <- ts(data[,2], frequency = 12, start = c(2019,6))

ts.plot(m)

```

```
dataNumeric <- as.numeric(unlist(data[,3]))
IPSA <- ts(data[,3], frequency = 12, start = c(2019,6))
ts.plot(IPSA)

dataNumeric <- as.numeric(unlist(data[,4]))
IR <- ts(data[,4], frequency = 12, start = c(2019,6))
ts.plot(IR)

dataNumeric <- as.numeric(unlist(data[,5]))
Pension <- ts(data[,5], frequency = 12, start = c(2019,6))
ts.plot(Pension)

m1 = lm(m~IPSA+IR+Pension)
summary(m1)
influenceIndexPlot(m1, vars="Cook")

par(mfrow=c(2, 2))
plot(m1, col='deepskyblue4', pch=19)

# yest<-predict(m1)
#
# plot(m1)
# abline(a=0, b=1, col="red")
```

TPMLillieforsMoran

```
    rm(list = ls())
rm()


#Lista de los paquetes a utilizar
packages <- c("foreign","apaTables","PerformanceAnalytics","psych","corrr","corrplot","pipeR",
              "dbplyr","ggplot2","factoextra","scales","cluster","NbClust","tidyverse","tidyr",
              "tidyverse","purrr","dplyr","FactoMineR","readr","dbscan","quantmod","dtw","fpc",
              "readxl","nortest","exptest")



#Funci n ipak para instalar y llamar a varios paquetes a la vez
ipak <- function(pkg){
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
```

```
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}

#Llamar los paquetes del listado anterior
ipak(packages)

#########
# DATOS #
#########

#Datos TPM
data <- read_excel("TPM.xlsx")
#dataNumeric <- as.numeric(unlist(data[,2]))
ts_data <- ts(data[,2], frequency = 12, start = c(2019,1))
ts.plot(ts_data, main = "Monetary Policy Rate (TPM) 2019-2022", xlab
   ="Date", ylab = "TPM")
grid()

### Normalidad todos los datos#######

# qqnorm(ts_data)
# qqline(ts_data)

shapiro.test(ts_data)
lillie.test(ts_data)

#### Exponencialidad todos los datos #########

moran.exp.test(ts_data)

###### Datos antes de constante (Jan2019 - Mar2020) #####

ts_data119_322 <- ts(data[1:15,2], frequency = 12, start = c(2019,1))
plot(ts_data119_322, main = "Monetary Policy Rate (TPM) Jan-2019-
   Mar2020", xlab ="Date", ylab = "TPM")
grid()

#### Normalidad #########

# qqnorm(ts_data119_322)
# qqline(ts_data119_322)

shapiro.test(ts_data119_322)
```

```
lillie.test(ts_data119_322)
skewness(ts_data119_322)
kurtosis(ts_data119_322)

#### Exponencialidad #########

moran.exp.test(ts_data119_322)

###### Datos despues constante (Jul2021 - Jan2022)#####

ts_data721_122 <- ts(data[31:37,2], frequency = 12, start = c(2021,7))
plot(ts_data721_122, main = "Monetary Policy Rate (TPM) Jul-2021-Jan2022", xlab ="Date", ylab = "TPM")
grid()

#### Normalidad #########

# qqnorm(ts_data721_122)
# qqline(ts_data721_122)

shapiro.test(ts_data721_122)
lillie.test(ts_data721_122)
skewness(ts_data721_122)
kurtosis(ts_data721_122)

#### Exponencialidad #########

moran.exp.test(ts_data721_122)
```