



# Properties of protons in nuclear medium with AdS/QCD model with a quadratic modified dilaton

Alfredo Vega<sup>1,a</sup>, Miguel Angel Martín Contreras<sup>1,b</sup>

<sup>1</sup> Instituto de Física y Astronomía, Universidad de Valparaíso, A. Gran Bretaña 1111, Valparaíso, Chile

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**Abstract** A modified version of the usual AdS/QCD soft wall model with quadratic dilaton is presented to study proton properties inside the atomic nucleus. We suggest a dilaton that catches the variation of the proton mass inside nuclei, called *nuclear dilaton*, and with this, we calculated electromagnetic form factors and magnetic moments for protons inside the atomic nucleus.

## 1 Introduction

It is well known that hadronic properties change inside the nuclear medium (see e.g., [1, 2]). Although studying these properties should consider QCD, its non-perturbative nature tends to make the calculations difficult, and this has motivated the development of several tools to address these theoretical studies of hadrons in this sector, such as lattice QCD (e.g., [3]), Schwinger–Dyson formulation (e.g., [4]), and the formulation of the phenomenological models that capture some of the hadronic constituent properties and make it possible to perform calculations. Among such models, we can distinguish those based on the AdS/CFT correspondence [5, 5–8].

The holographic dual of QCD is currently unknown, but we can use the AdS/CFT idea to build bottom-up phenomenological models that simulate some of the most important attributes of hadrons (these approaches are called AdS/QCD models). A close exploration of the specialized literature reveals that AdS/QCD models have been successfully applied to a wide range of hadronic phenomenology. For example, hadrons in vacuum, hadrons at zero temperature (e.g., see [9–22]), at finite density (e.g., see [23–29]) and at finite temperature hadrons (e.g., see [30–35]). In these models, *in-medium* properties are enclosed into a metric that defines the space where modes duals to hadrons live.

From the gravitational point of view, the metric and the dilaton field define the background. Both are related dynamically as solutions of the equations of motion obtained from the Hilbert–Einstein action variations [36–42]. However, in the phenomenological AdS/QCD models context, it is common to use static dilatons (which are not dynamically related to the metric, e.g., see [9–22]) to softly break the conformal symmetry. In general, the dilaton is not considered a part of the background that could catch medium properties affecting the hadrons studied in these models.

An AdS/QCD model that considers the nuclear medium effects on the hadronic properties has been discussed in [43], where authors have considered additional fields in the bulk, leading to a specific metric called *thermal charged AdS* (tcAdS) [44–46]. Using this proposal, it was possible to address some hadronic properties in medium [28, 29, 47–50].

The approach developed here is different, and it is motivated by three phenomenological aspects that can be implemented consistently in an AdS/QCD model with a quadratic dilaton if one keeps open to the possibility that, as being part of the background, the dilaton catches properties of the medium and complement the information enclosed in the metric. First, by looking at the QCD phase diagram, at low temperatures, hadrons in vacuum and hadrons inside the nucleus are in the same region [51]. This implies that the holographic formulation of in-medium hadrons could be addressed using asymptotically AdS spaces, like those used in AdS/QCD soft wall models. Therefore, the medium properties could be enclosed in the other background ingredient at hand: the dilaton field. In previous work, we considered this idea in the context of finite temperature introducing a *thermal dilaton*, which makes it possible to modify the melting temperature of hadrons inside a thermal bath [52]. We consider it could be useful to improve the AdS/QCD models at finite temperature, as can be seen in a recent work where authors consider thermal dilatons to study the dependence

<sup>a</sup> e-mail: [alfredo.vega@uv.cl](mailto:alfredo.vega@uv.cl) (corresponding author)

<sup>b</sup> e-mail: [miguelangel.martin@uv.cl](mailto:miguelangel.martin@uv.cl)

on temperature of masses and form factors in mesonic [53] and baryonic [54] sectors.

Second, hadron masses, which in medium vary with the density of the medium according to specific scaling rules [55–59] which can be implemented easily in the quadratic dilaton context and by considering a small change in the scalar field used to break the conformal invariance in AdS side.

Finally, the currents that define the nucleon form factors, in the vacuum and nuclear medium, have the same mathematical structure [60]. This suggests that, on the AdS side, the interaction terms that give rise to the form factors also have the same structure. Therefore, the expressions for the in-medium form factors in terms of the AdS modes should be similar to those in AdS/QCD models in the vacuum.

In this work, following the three previous points exposed above, we propose a simple extension of a model which considers an AdS metric and a quadratic dilaton to study proton electromagnetic properties. As in much of the soft wall model approaches, we consider a static dilaton whose functional form is chosen to reproduce the observed mass spectrum. In this work, we follow the scaling rule given in [55–59] to relate hadron masses in the vacuum with its masses in medium, which in the case of the holographic models with a quadratic dilaton, can be implemented in a simple form. It is enough to change the parameter used to define a dilaton that depends on the density, which we call a *nuclear dilaton*. This new dilaton captures some of the properties of the nuclear medium we are interested in, and we use it with expressions reported in [61, 62] to study electromagnetic properties of nucleons, obtaining good results.

This work is structured as follows: in Sect. 2, we show the phenomenological framework that led us to the modifications of the holographic model considered, namely the QCD phase diagram, the scaling rules, and the proton form factors. In Sect. 3, we present a summary of procedures presented in [61, 62] to calculate nucleon electromagnetic form factors, and in Sect. 4, we discuss how to modify it according to the phenomenology discussed in section II. Finally, in Sect. 5 we give conclusions and final remarks about this work.

## 2 Phenomenological background

In this section, we will show the phenomenological background that motivates us to suggest how the dilaton can be modified to capture the medium properties where the hadrons live. We will focus on protons.

### 2.1 Nucleons and QCD phase diagram

By observing the QCD phase diagram [51], it is possible to note that, at low temperatures and densities close to the nuclear medium, nucleons live inside atomic nucleus or free

live in the same region, and they do not experience phase transitions.

These lead us to explore the possibility of studying hadrons in the nuclear medium with the AdS/QCD models, where the metric used to study them is the same as the one employed in the vacuum. This suggests that nuclear medium properties are captured in a different background field, the dilaton.

### 2.2 Hadron masses in nuclei

From the middle of the eighties is known that hadron properties change in medium, and especially we learn that hadron mass inside the atomic nucleus vary according to scaling rules as [55–59], which in the range of densities between zero and the nuclear density  $\rho_0$ , scales linearly as

$$\frac{M^*}{M} = 1 - C \frac{\rho_B}{\rho_0}, \quad (1)$$

where  $M^*$  is the hadron mass in the nuclear media,  $M$  is its mass in the vacuum,  $\rho_B$  is the medium density,  $\rho_0$  is the density in the nucleus, and  $C$  is a constant (here we consider  $C = 0.21$ , which is the value calculated in [59] for protons in the quark–meson coupling (QMC) model).

This scaling observed for hadron masses in this paper is considered a guideline that can be implemented straightforwardly with a small change in AdS/QCD models with quadratic dilaton.

### 2.3 Electromagnetic form factors of nucleons inside the nucleus

Nucleon electromagnetic form factors  $F_1^N$  and  $F_2^N$  ( $N = p, n$  labels proton and neutron) are conventionally defined by the matrix elements of the electromagnetic current  $J_{EM}^\mu$  as

$$\langle p' | J_{EM}^\mu(0) | p \rangle = \bar{u}(p') [\gamma^\mu F_1^N(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2^N(Q^2)] u(p), \quad (2)$$

where  $q = p' - p$  is the momentum transferred,  $m_N$  is the nucleon mass,  $F_1^N$  and  $F_2^N$  are the Dirac and Pauli form factors, normalized to the electric charge  $e_N$  and the anomalous magnetic moment  $k_N$  of the corresponding nucleon, i.e.:  $F_1^N(0) = e_N$  and  $F_2^N(0) = k_N$ .

Using the Dirac and Pauli form factors, it is possible to build the so-called *Sach electric and magnetic form factors* as

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \quad (3)$$

and

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2). \quad (4)$$

At  $Q^2 = 0$ , the last Sach form factor defines the magnetic moment in nuclear magneton units, i.e.  $G_M(0) = \mu$ , that in proton case is related to anomalous magnetic moment by  $\mu = 1 + k_p$ .

In order to consider medium effects, we use a star (\*) to establish a difference with quantities in the vacuum. Assuming that a baryon is quasi-free in the nuclear medium, the electromagnetic current for protons can be expressed as [60]

$$\langle p' | J_{EM}^\mu(0) | p \rangle = \bar{u}(p') [\gamma^\mu F_1^{N*}(Q^2) + \frac{i\sigma^{\mu\nu}}{2m_N^*} q_\nu F_2^{N*}(Q^2)] u(p), \tag{5}$$

where  $F_1^{N*}$  and  $F_2^{N*}$  are the Dirac and Pauli form factors in nuclear medium, which are normalized to electric charge  $e_N$  and anomalous magnetic moment  $k_N$  of the corresponding nucleon:  $F_1^{N*}(0) = e_N$  and  $F_2^{N*}(0) = k_N^*$ .

As in the vacuum, we define the electric and magnetic form factors as

$$G_E^*(Q^2) = F_1^*(Q^2) - \frac{Q^2}{4(M^*)^2} F_2^*(Q^2) \tag{6}$$

and

$$G_M^*(Q^2) = [F_1^*(Q^2) + F_2^*(Q^2)] \frac{M}{M^*}. \tag{7}$$

An additional factor was included in the last expression to change the magnetic moments into nuclear magneton units to compare with the vacuum results easily.

The structure of the currents that define the electromagnetic form factors is the same in both cases. Thus, the interaction terms in the AdS side that give rise to such electromagnetic form factors should also have the same structure at the nuclear medium. Therefore, following these observations make it possible to adapt the existent models at zero temperature and zero density to perform calculations for the hadronic properties in the nuclear medium.

### 3 Soft wall model for nucleons in vacuum

The calculation of hadronic electromagnetic form factors in AdS/QCD models depends on the interaction terms present in the action. For nucleons in a vacuum, it was considered in [63]. Later, it was suggested a model [61, 62] that considers a more general interaction term in the AdS side to study electromagnetic and axial properties in nucleons. This approach additionally considers different Fock states to describe nucleons in the AdS side.

In order to do self-consistent our work, in this section, we do a small summary of the model proposed in [61] to describe nucleons in the vacuum, and in the next section, we will adapt it to describe nucleon properties in a nuclear medium.

Our geometric background is set by the usual AdS<sub>5</sub> Poincare chart defined as

$$dS^2 = e^{2A(z)} [dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu], \tag{8}$$

where  $A(z) = \log(R/z)$  for AdS-like geometries.

By simplicity here, we reproduce the relevant part to calculate proton electromagnetic form factors only. In this case, the AdS/QCD action considers fermionic fields  $\Psi_{\pm,\tau}(x, z)$  with spin 1/2 (dual to nucleons) with scaling dimension  $\tau$  and a vector field  $V_M(x, z)$  with spin 1 (holographically equivalent to the electromagnetic field).

The action considered has the following form

$$S = \int d^4x dz \sqrt{g} e^{-\phi(z)} \{ \mathcal{L}_\Psi(x, z) + \mathcal{L}_V(x, z) + \mathcal{L}_{int}(x, z) \}, \tag{9}$$

$$\mathcal{L}_\Psi(x, z) = \sum_{i=+,-} \sum_{\tau} c_\tau \bar{\Psi}_{i,\tau} \mathcal{D}_i \Psi_{i,\tau}, \tag{10}$$

$$\mathcal{L}_V(x, z) = -\frac{1}{4} V_{MN}(x, z) V^{MN}(x, z), \tag{11}$$

$$\mathcal{L}_{int}(x, z) = \sum_{i=+,-} \sum_{\tau} c_\tau \bar{\Psi}_{i,\tau} \gamma_i \Psi_{i,\tau}, \tag{12}$$

where

$$\mathcal{D}_\pm = \frac{i}{2} \Gamma^M \overleftrightarrow{\partial}_M \mp (\mu_5 + U_F(z)) \tag{13}$$

$$\mathcal{V}_\pm(x, z) = Q \Gamma^M V_M \pm \frac{i}{4} \eta_\nu [\Gamma^M, \Gamma^N] V_{MN}(x, z) \pm g_V \tau_3 \Gamma^M i \Gamma^z V_M(x, z). \tag{14}$$

Here  $V_{MN} = \partial_M V_N - \partial_N V_M$  is the stress tensor of the vector field,  $Q = \text{diag}(1, 0)$  is the nucleon charge matrix,  $\tau_3 = \text{diag}(1, -1)$  is the Pauli isospin matrix,  $A \overleftrightarrow{\partial} B = A(\partial B) - (\partial A)B$ ,  $\phi(z) = \kappa^2 z^2$  is the static dilaton field dilaton field with  $\kappa$  as a free energy scale [9],  $\Gamma^M = \epsilon_a^M \Gamma^a$  and  $\Gamma^a$  re the five dimensional Dirac matrices. The quantity  $\mu_5$  is the bulk fermion mass related to scaling dimension  $\tau$  via the expression  $m = \mu_5 R = \tau - 3/2$ . The scaling dimension of the baryon interpolating operator is related to the paron number  $N$  by  $\tau = N + L$ , and  $L$  is the maximum value of the  $z$  component of quark orbital angular momentum in the light front wave function (along this paper we restrict us to case  $L = 0$ ). In the model also was considered  $U_F(z) = \phi(z)/R$ .

The fields  $\Psi_\tau$  describe nucleon Fock states with different number of constituents in the AdS side, and  $c_\tau$  are a set of parameters constrained by  $\sum_\tau c_\tau = 1$ . Rescaling fermionic fields as

$$\Psi_{i,\tau}(x, z) = e^{\phi(z)/2} \psi_{i,\tau}(x, z), \tag{15}$$

we can obtain the E.O.M. for  $\psi_{i,\tau}(x, z)$ . Additionally, if we split this fermionic field into left and right components

$$\psi_{i,\tau}(x, z) = \psi_{i,\tau}^L(x, z) + \psi_{i,\tau}^R(x, z), \tag{16}$$

and perform a KK expansion for chiral components

$$\psi_{i,\tau}^{L/R}(x, z) = \frac{1}{\sqrt{2}} \sum_n \psi_{i,\tau}^{L/R}(x) F_{i,\tau,n}^{L/R}(z), \tag{17}$$

where  $\psi_{i,\tau}^{L/R}(x)$  are four dimensional fields on boundary. With this it is possible to obtain two coupled one dimensional EOMs for  $F_{i,\tau,n}^{L/R}(z)$ , and finally, with an additional substitution ( $F_{i,\tau,n}^{L/R}(z) = e^{-2A(z)} f_{i,\tau,n}^{L/R}(z)$ ) it is possible obtain the EOM for fields which we use to calculate electromagnetic form factors.

For models considering AdS metrics as (8), altogether with quadratic static dilaton  $\phi(z) = \kappa^2 z^2$ , it is possible to get a Schrodinger-like equation for the chiral spinors as

$$\left[ -\partial_z^2 + \kappa^4 z^2 + 2\kappa^2 \left( m \mp \frac{1}{2} \right) + \frac{m(m \pm 1)}{z^2} \right] f_{\tau,n}^{L/R}(z) = M_{n\tau}^2 f_{\tau,n}^{L/R}(z), \tag{18}$$

where

$$f_{\tau,n}^L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}} \kappa^\tau z^{\tau-1/2} e^{-\frac{\kappa^2 z^2}{2}} L_n^{\tau-1}(\kappa^2 z^2), \tag{19}$$

$$f_{\tau,n}^R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\frac{\kappa^2 z^2}{2}} L_n^{\tau-2}(\kappa^2 z^2), \tag{20}$$

which are normalized and

$$M_{n\tau}^2 = 4\kappa^2(n + \tau - 1). \tag{21}$$

Integrating over the holographic coordinate and constraining the action to obtain a four-dimensional action for fermions, it is possible to identify that nucleon masses correspond to

$$M_n = \sum_\tau c_\tau M_{n\tau} = 2\kappa \sum_\tau c_\tau \sqrt{n + \tau - 1}, \tag{22}$$

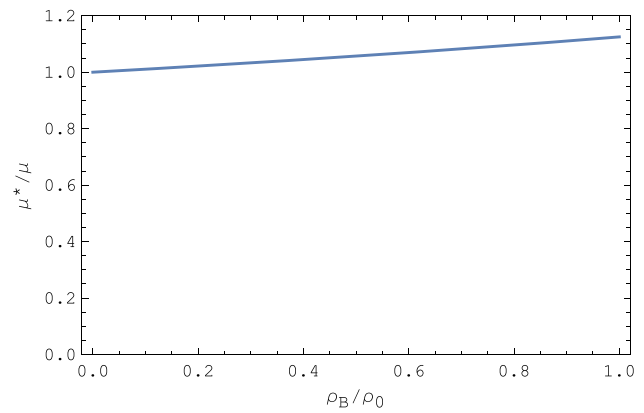
and  $c_\tau$  must satisfy  $\sum_\tau c_\tau = 1$ .

On the other side, by analyzing the term  $\mathcal{L}_V$ , it is possible to obtain the bulk to boundary propagator  $V(Q, z)$  whose convenient representation for form factor calculations is [64]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)}, \tag{23}$$

where  $x$  is equivalent to light cone momentum fraction [11].

Finally, from the interaction term in the action that considers minimal and non-minimal couplings of the fermion and vector field in AdS, it is possible to obtain the electromagnetic form factors. For protons are calculated by evaluating



**Fig. 1** The magnetic moment as a function of the in-medium density divided by the proton magnetic moment in vacuum

the following expressions:

$$F_1(Q^2) = C_1(Q^2) + g_v C_2(Q^2) + \eta_V^p C_3(Q^2) \tag{24}$$

and

$$F_2(Q^2) = \eta_V^p C_4(Q^2), \tag{25}$$

where  $Q^2 = -t$  and  $C_i(Q^2)$  correspond to integrals given by

$$C_1(Q^2) = \frac{1}{2} \int dz V(Q, z) \sum_\tau c_\tau \{ [f_{L\tau}(z)]^2 + [f_{R\tau}(z)]^2 \} \tag{26}$$

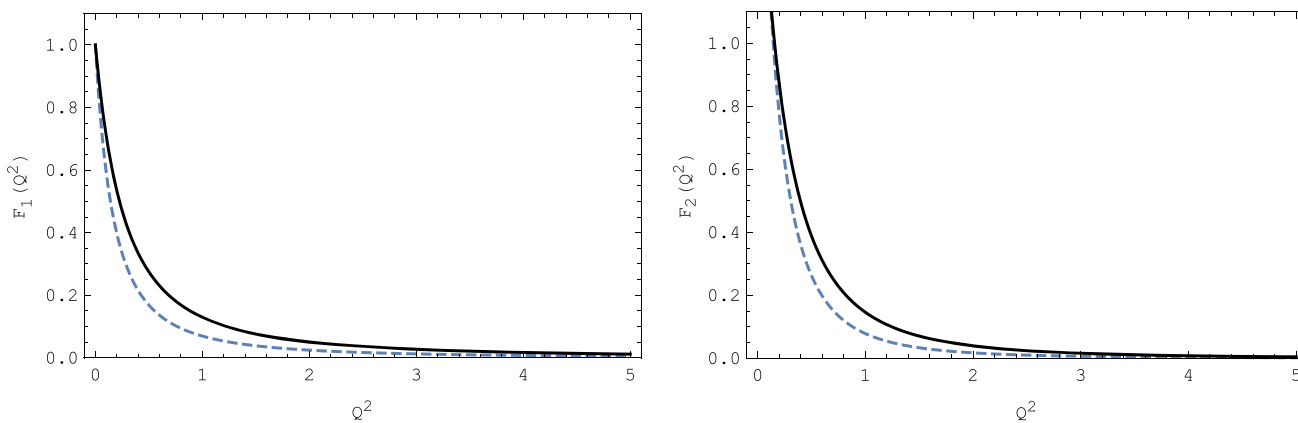
$$C_2(Q^2) = \frac{1}{2} \int dz V(Q, z) \sum_\tau c_\tau \{ [f_{L\tau}(z)]^2 - [f_{R\tau}(z)]^2 \} \tag{27}$$

$$C_3(Q^2) = \frac{1}{2} \int dz z \partial_z V(Q, z) \sum_\tau c_\tau \{ [f_{L\tau}(z)]^2 - [f_{R\tau}(z)]^2 \} \tag{28}$$

$$C_4(Q^2) = 2M \int dz z V(Q, z) \sum_\tau c_\tau \{ [f_{L\tau}(z)]^2 [f_{R\tau}(z)]^2 \}, \tag{29}$$

#### 4 Soft wall model for the nuclear medium

In [61–63], the authors use AdS/QCD soft wall-like models with an AdS metric and a quadratic dilaton to calculate proton electromagnetic form factors. Both models can be extended in an easy way to study properties of protons inside nuclei, and the procedure could also be extended to other hadrons with these sorts of holographic models. In this work, we consider the model discussed in [61], which allows studying nucleon structure, including high Fock states in AdS/QCD, but by simplicity restrict ourselves to the valence case. For



**Fig. 2** The graphics show the Dirac and Pauli form factors for protons when  $\rho_B/\rho_0 = 0$  (continuous line) and  $\rho_B/\rho_0 = 1$  (dashed line)

this reason, we use expressions calculated in detail in [61], with  $c_3 = 1$  and  $c_i = 0$  in other cases.

According to [61] for the valence case, the proton radial mass spectrum in the vacuum is given by:

$$M_n^2 = 4 \kappa^2 (n + 2). \tag{30}$$

Now let us extend these ideas to the finite density case in order to address nuclear matter systems. Holographically speaking, these sort of finite density systems are described by two geometries depending if they are confining or not. The deconfined case is dual to a Reissner–Nordstrom AdS black hole. In the confined case, the geometry is the so-called *thermal charge* AdS space [46], defined by

$$dS_{\text{Th-Q}}^2 = e^{2A(z)} \left[ \frac{dz^2}{h(z)} - h(z) dt^2 + d\mathbf{x} \cdot d\mathbf{x} \right], \tag{31}$$

which describe a medium at zero temperature with finite density, where  $h(z) = 1 + q^2 z^6$ , where  $q$  is the  $U(1)$  charge dual to the chemical potential, and therefore, responsible for capturing the medium effects into the geometry, and according to [46], when  $N_c = 3$  and  $N_f = 2$  are considered, we obtain for the normal nuclear density  $q \sim 0.05$ . Recall that when  $q$  goes to zero, we recover (8), as in our studied case. Therefore, to capture the medium effects, since these sorts of bulk perturbations in the metric can be neglected, we propose to include them phenomenologically in the dilaton field.

A proposal is to use the static quadratic dilaton to address in-medium properties as temperature [52–54] and finite density. In the latter case, this idea is supported by the well-known scaling, in which hadron masses scale with the ratio between medium and nuclear densities. In particular, for the proton mass, we consider the following relation according to QMC model [59]

$$M_n^* = \left( 1 - 0.21 \frac{\rho_B}{\rho_0} \right) M_n, \tag{32}$$

which works for  $\rho_B/\rho_0$  between zero and one, and the star (\*) quantities are defined *in-medium*. This fact suggests us to scale the holographic proton mass spectrum, given in equation (30), into a similar form

$$M_n^{*2} = 4 \kappa^{*2} (n + 2), \tag{33}$$

with the dilaton slope re-scaled as

$$\kappa^* = \sqrt{\left( 1 - 0.21 \frac{\rho_B}{\rho_0} \right)} \kappa. \tag{34}$$

This allows to reproduce the scaling behaviour for the proton mass inside the nuclei, as it was suggested in [55–59]. Its use in AdS/QCD predicts a Regge behavior for hadrons inside atomic nuclei according to eq. (1).

Considering that our model is restricted to low densities if the masses in vacuum are ordered in Regge trajectories, this behavior is expected to be preserved at the explored region. In fact, the nucleon masses are shifted from the vacuum values in a small amount. Therefore, we believe this could also happen in the nucleon excited states. Thus, the existence of a Regge trajectory, like (33), is reasonable.

The latter suggests to us that the effect of the remaining nucleus on the considered hadron can be addressed in a soft wall-like model using a different dilaton than used to study hadrons in the vacuum, that we call *nuclear dilaton* since it captures some properties of the nuclear medium where the hadron is living.

As we mention at the beginning of this section, we consider the AdS/QCD model discussed in [61] to calculate the nucleon electromagnetic form factors, and then we adapt it to study protons in the nuclear medium.

As was discussed in Sect. 2.3, the electromagnetic current in the vacuum and low-density media are similar. Therefore, interaction terms in AdS side used to extract form factors must contain the same structure used in [61] to describe protons in the vacuum, and expressions for form factors, in this case, must be similar to those calculated in this paper, with



the only change that  $\kappa \rightarrow \kappa^*$ , i.e.,

$$F_1^*(Q^2) = C_1^*(Q^2) + g_v C_2^*(Q^2) + \eta_V^p C_3^*(Q^2) \quad (35)$$

and

$$F_2^*(Q^2) = \eta_V^p C_4^*(Q^2). \quad (36)$$

In this work, we consider only the contribution coming from the valence part, so we used the expressions given in previous section [61], considering  $\tau = 3$ . This leads us to functions  $C_i^*(Q^2)$ , which are given by

$$C_1^*(Q^2) = \frac{1}{2} \int dz V^*(Q, z) \{ [f_L^*(z)]^2 + [f_R^*(z)]^2 \} \quad (37)$$

$$C_2^*(Q^2) = \frac{1}{2} \int dz V^*(Q, z) \{ [f_L^*(z)]^2 - [f_R^*(z)]^2 \} \quad (38)$$

$$C_3^*(Q^2) = \frac{1}{2} \int dz z \partial_z V^*(Q, z) \{ [f_L^*(z)]^2 - [f_R^*(z)]^2 \} \quad (39)$$

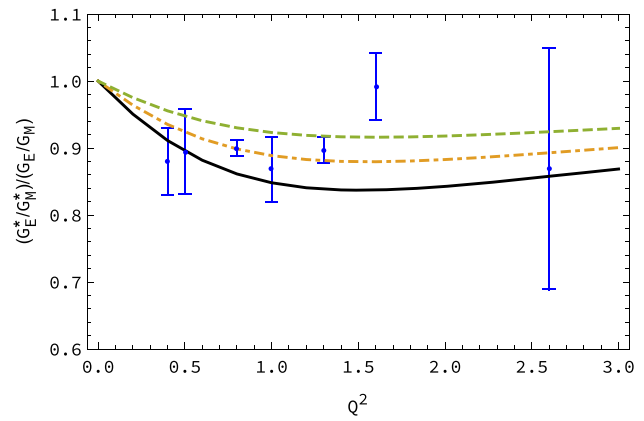
$$C_4^*(Q^2) = 2M^* \int dz z V^*(Q, z) \{ [f_L^*(z)]^2 [f_R^*(z)]^2 \}, \quad (40)$$

where we used the  $\kappa^*$  parameter for the nuclear medium to calculate  $V^*(Q, z)$  and  $f_{L/R}^*(z)$ .

By considering the part with  $\tau = 3$  only, some of the parameters changed with respect to those used in [61]. Throughout this paper we take  $g_v = 0.3$  as in [61], but we consider  $c_3 = 1$ , and other  $c_\tau = 0$  (in agreement with condition after equation (22) for one  $\tau$ ). The parameter  $\eta_V^p = 0.224$  is fixed when we take  $G_M(0)$  in order to obtain the magnetic moment for protons in vacuum. Recall that  $G_M(0) = \mu_{\text{proton}} = 2.793$ .

Following the discussion above, the values of the magnetic moment and the charge for the proton fix the form factors in the vacuum. Then, by implementing the holographic scaling rule in eqn. (34) we construct the proton form factors in-medium with the same set of  $\eta_V$  and  $c_\tau$  as the one used in the vacuum. These results are displayed in Fig. 1, which shows proton magnetic moment as a function of the in medium density ( $\mu^* = G_M^*(0)$ ) divided magnetic moments for protons in the vacuum and Fig. 2 that shows Dirac and Pauli form factors in the vacuum and nuclear medium according to eqns. (35) and (36).

In order to compare our results, we consider the measurements made at JLab [65,66] and Mainz Microtron (MAMI) [67] for polarization-transfer double ratio for a bound proton. These data can be connected with the electromagnetic form factor (EMFF) double ratio  $(G_E^*/G_M^*)/(G_E/G_M)$ , showing this quantity is not equal to one. Consequently, it implies medium modifications to the bound proton electromagnetic form factors, and additionally, as several authors have calculated by different theoretical approaches. In comparison to other calculations, our results are qualitatively similar to



**Fig. 3** Double ratio  $(G_E^*/G_M^*)/(G_E/G_M)$  for protons. Densities considered are  $\rho_B/\rho_0 = 1$  (continuous line),  $\rho_B/\rho_0 = 0.8$  (dot-dashed line) and  $\rho_B/\rho_0 = 0.6$  (dashed line). Experimental data for protons are from Refs. [65–67]

those presented in [68], based on the chiral quark-soliton (CQS) model. We conclude this because, in both approaches, the double ratio is one when  $Q^2 = 0$  and presents a minimum in the plotted region. However, in [68], the minimum is slightly deep and is located at values slightly higher for  $Q^2$ . Other calculations that present a minimum for the double ratio are based on the quark-meson coupling (QMC) model, e.g., [69]. Nevertheless, the double ratio at  $Q^2 = 0$  is lower than zero. A recent calculation [70] that considers all of the members in the baryon octet as symmetric nuclear matter in the range  $Q^2 = 0$  to  $3 \text{ GeV}^2$ , shows for protons a double ratio that decreases. A behavior almost constant for the double ratio is predicted in a relativistic light-front constituent model [71].

### 5 Conclusions

In AdS/QCD models, the properties of hadrons in dense media or finite temperature, in general, consider dealing with AdS black hole (AdS-BH) geometries. In the case of temperature only, we have some possible solutions: thermal AdS for the confined phase, AdS-Schwarzschild black hole in the deconfined phase [72], and there is also the *thermal charged AdS* (tcAdS) [44–46], which is sometimes used to calculate properties of hadrons in nuclear media. In all of these cases, medium properties are encoded exclusively in the metric.

Although dilatons used in most AdS/QCD models are static, it should be remembered that they are background fields (which in dynamical models are related or coupled to metric). Motivated by this fact, we suggest they could complement the metric tensor capturing part of the medium properties. Thus, they can help us describe hadron properties in a medium and/or finite temperature.

To the best of our knowledge, the only attempt to incorporate part of the medium properties in a dilaton was made in [52], where the authors consider a thermal dilaton, i.e., a dilaton that depends on temperature also, showing that it is possible to modify the melting hadron temperatures with this kind of dilatons. In recent work, this type of dilaton was implemented to study the dependence on temperature of masses and form factors in the mesonic [53], and baryonic [54] sectors.

In this paper, we explore the same idea to study proton properties inside the atomic nucleus. We consider a modified dilaton considering three phenomenological aspects that suggest a simple modification of the AdS/QCD models with AdS-like metrics and a quadratic and static dilaton. As it was illustrated in Figs. 1 and 2, this yields good quantitative results. The hadron mass is increased with nuclear density, magnetic moments are reduced, and the electromagnetic form factors follow a similar behavior obtained with other approaches.

In Fig. 3 we show our results for the double ratio  $(G_E^*/G_M^*)/(G_E/G_M)$  at different densities and we compare with experimental data [65–67]. Our results for  $Q^2 < 3.0 \text{ GeV}^2$  shows a similar behaviour as in the chiral quark soliton model discussed in [68].

Our methodology can be summarized as follows: we have extended the vacuum holographic calculations associated with the proton form factor to the nuclear medium by considering how the dilaton slope scales with the nuclear density. The start point is the form factor at the vacuum that sets the parameter  $\eta_V$ . The proton mass at vacuum fixes  $c_\tau$ . Then, apply the holographic scaling rule, eqn. (34), to compute the in-medium proton form factor. This procedure is an alternative to the standard thermal-charge AdS metric used with finite density applications before the deconfinement phase transition.

In future studies, we contemplate delving more deeply into other proton properties in the nucleus with the extension suggested in this paper and study the properties of other hadrons inside the nuclear medium.

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