Received 9 March 2019
Revised 29 April 2019
Accepted 26 June 2019

# A joint replenishment supply model for multi-products grouped by several variables with random and time dependence demand

Fernando Rojas
*Centro de Mico-Bio Innovación, Escuela de Nutrición y Dietética, Facultad de Farmacia, Universidad de Valparaíso, Chile*

**Abstract**

**Purpose** – This paper aims to propose a supply model of periodic review with joint replenishment for multi-products grouped by several variables with random and time dependence demand.

**Design/methodology/approach** – The products are grouped by multivariate cluster analysis. The stochastic inventory model describes the random demand of each product, considering the temporal dependency through a generalized autoregressive moving average model. Stochastic programming for the total cost of inventory is obtained considering the expected value of the demand per unit of time.

**Findings** – The total costs for the products grouped with the proposed model are 6% lower than for the individual inventory policy. The expected shortage units decrease significantly in the proposed grouped model with temporary dependence. In addition, the proposal with temporary dependency has lower costs than when the independent and identically distributed demand is considered.

**Originality/value** – The proposed policy is exemplified with real-world data from a Chilean hospital, where the products (drugs) are segmented by grouping variables, forming clusters of drugs with homogeneous behavior within the groups and heterogeneous behavior between groups.

**Keywords** Time series, Inventory management, Management information systems, Supply chain management, Stochastic programming, Cluster analysis, GARMA model, Joint replenishment, Periodic review

**Paper type** Research paper

## 1. Introduction

It has been shown that supply systems reduce the vulnerability of supply chain management in companies. This reduction is achieved by optimizing the inventory levels necessary to meet customer demand for components and final products offered by companies (Hillier and Lieberman, 2005).

Originally, the inventory models based on periodic review consider the demand of the products in a deterministic way (Atkins and Iyogun, 1988b). Subsequently, the uncertainty in demand is addressed through of stochastic inventory models of periodic review that consider the demand for products as a random variable (RV) described by a continuous or discrete distribution (Lee and Lee, 2013). This change to a framework of stochastic programming to solve supply problems, is critical for inventory management through periodic review and joint replenishment in stochastic demand environments (Eynan and Kropp, 1998). The total costs (TC) of placing an order to the supplier for a number of different products have two components:

(1) a major ordering cost independent of the number of different products in the order; and

Journal of Modelling in Management
Vol. 15 No. 1, 2020
pp. 276-296

1746-5664
DOI 10.1108/JM2-03-2019-0061

(2) a minor ordering cost which depends on the number of different products in the order.

In the joint replacement problem, a family of items is purchased from a single provider (Eynan and Kropp, 1998). To achieve savings in TC of inventory, multiple products grouped can be replenished jointly with the same cycle of periodic review for all assortment of inventory (Khouja and Goyal, 2008). Van Eijs *et al.* (1992) have proposed algorithms to find solutions for the joint replenishment problem (JRP). Strategies to solving the JRP can be classified into two types: A direct grouping strategy (DGS) and an indirect grouping strategy (IGS). Under DGS, products are partitioned into a predetermined number of sets and the products within each set are jointly replenished with the same cycle time. Under IGS a replenishment is made at regular time-intervals and each product has a replenishment quantity sufficient to last for exactly an integer multiple of the regular time interval. Groups in IGS are indirectly formed by products having the same integer multipliers.

Multivariate analysis of cluster proposes a way to accomplish groupings objects, considering the minimum distance of each with respect to multiple variables that can serve to characterize the groups to form (Johnson and Wichern, 2007). Although Tsai *et al.* (2009) described the use of cluster analysis to be occupied as DGS in JRP, this technique has not been used to include multiple variables that could be important in the characterization of an assortment of inventories to be grouped. In this case, we propose multivariate cluster analysis to use DGS, characterizing the conformed product groups in JRP.

Once indicators of these inventory have been defined and distributional assumptions for demand per unit of time (DPUT) and for demand during lead-time (LT), in short LTD, have been established, the expected value of the objective function based on the inventory TC must be optimized (Namit and Chen, 1999). Often RVs DPUT are not independent and identically distributed (IID) over time, see Kristianto *et al.* (2012) and Rojas *et al.* (2019). When you want to optimize inventory composed of multiple products, should not be omitted the possible time dependence of the demand for products that conform the assortment of inventory (Calfa, 2015). Such optimization can be improved if the said time dependence is considered by conditional distributions to last information, considering a generalized linear autoregressive moving average models (GARMA) (Benjamin *et al.*, 2003), as through it is possible to obtain the parameters that will require stochastic programming of inventory models of periodic review in JRP.

The objective of this work is to propose a methodology based on inventory models for JRP for multiple products grouped by several variables, for supply from a single distributor. The presentation of this research is organized as follows: Section 2 makes a literature review about:

- modeling DPUT of products conditionally to past information based on the GARMA model;
- inventory models of periodic review for JRP based in cluster analysis;
- stochastic programming; and
- problem to solve through of the inventory policy propose.

Section 3 exposes the proposed methodology, whereas Section 4 illustrates it taking drugs supply of a Chilean public hospital with real-world data; and Section 5 provides a discussion of the results obtained in this research, as well as its limitations and future research.

## 2. Literature review

### *2.1 Modeling DPUT conditional to past information with GARMA model*

Often RVs DPUT are not independent and identically distributed (IID) over time, see Calfa (2015). It could also be the case of statistical dependence between products that make up the inventory assortment, in which case it is possible to model the demand with conditional probability distributions, see Rojas and Leiva (2016). Both dependences can be modeled by autoregressive and moving average (ARMA) time series (Kristianto *et al.*, 2012). GARMA models consider ARMA components by transforming the mean of the data via a link function in the line of generalized linear models (GLM) (Benjamin *et al.*, 2003). In addition, GARMA models allow for other distributions different from the normal one, which has the disadvantage of predicting negative values when the data take very low values or even zeros (Prabakaran *et al.*, 2013). Furthermore, GARMA models are widely flexible, easy to estimate and interpret, and besides its prediction is straightforward (Dunsmuir and Scott, 2015). Although it is possible to obtain normality by a logarithmic transformation for the data, it cannot be used if zero values are present in the data, as usual when modeling demand data. Prediction based on GARMA models may be carried out by the density forecast (DF) technique; for more details, see Diebold *et al.* (1998), Diebold *et al.* (1999), Bauwens *et al.* (2004) and Calfa (2015). As DPUT can show time dependence, there exists a need to propose inventory models involving this temporal dependence, see Gilbert (2005) and Rojas *et al.* (2019). Such a temporal structure may be added into the inventory costs as part of the objective function or into its constraints. Due to the stochastic nature of the serial dependence of the DPUT values to be modeled (in this case the time conditional DPUT), an important contribution of our paper will be to take advantage of the flexibilities regarding to statistical distribution used to model DPUT, its easy interpretation and parameter estimation that justify the use of GARMA model to describe the DPUT in case of existence of temporary dependence of this random variable, to later occupy its expected value in a stochastic programming of the JRP.

### *2.2 Inventory models of periodic review for joint replenishment problem based in cluster analysis*

A problem of multiple products commonly occurring in inventory policies is to decide what optimal quantities of products must be ordered simultaneously from a same supplier, see Pasandideh *et al.* (2018). The cost of placing an order for multi-products has two parts:

(1) one is independent of the amount of units of the products to be ordered (major ordering cost); and

(2) another depending on items to order (minor ordering cost).

This is referred to as JRP and considers the savings obtained with an inventory policy whose major ordering cost of multiple products to a supplier is higher than their minor ordering cost of items. Many algorithms have been proposed to find solutions for the JRP (Van Eijs *et al.*, 1992). However, when supply is a single distributor the major ordering cost for a group of products is equal to a minor ordering cost of one single product. In this work, we will propose a way to group articles to solve JRP, such that it allows us to generate a homogenous profile of the products to be grouped, and that in turn considers that in many cases the DPUT must be described considering a temporal sequence model, so we will compare our proposal regarding the non-grouping of items (individual replenishment of products) and regarding the non-consideration of a temporary model to describe the DPUT of the items to be grouped.

The deterministic JRP has been studied by many researchers. Atkins and Iyogun (1988a) have considered the case where demand varies in the time by extending the Silver (1973) heuristic, but always under a deterministic approach. The savings potential using JRP is also viable for stochastic demand. However, in such environments coordination and control are more difficult (Eynan and Kropp, 1998). Originally, Balintfy (1964) proposed simple control rules, where all items are continuously monitored, and it is possible to take advantage of the investment in the order of an item to raise the level of another according to its stochastic demand behavior. Atkins and Iyogun (1988b) reported that the best stochastic solutions for the JRP are given by the periodic review approach (except for small order cost values a set of items). Eynan and Kropp (1998), examined the periodic review system under stochastic demand and showed that the sequential approach that is commonly used can result in substantial penalties for obtaining optimal costs, since the sequential approach ignores the relationship between cycle time and the required safety stock, proposing a simple solution procedure, which nevertheless provides almost optimal solutions. Always under uncertain demand, Braglia *et al.* (2016) studied JRP, with backorders-lost salts mixtures, controllable lead times, and investment to reduce the major ordering cost. The authors propose an efficient and more practically applicable solution procedure, approximating part of the cost function with its second-order Taylor series expansion, obtained an expression that resembles the deterministic cost structure. Therefore, the problem can be approached exploiting a standard algorithm suitable for the deterministic JRP. It should be noted that controlling both the lead-time and the cost of order/setup is a major problem in the stochastic case of inventory replenishment and a key objective of the Just-in-time philosophy. Many benefits can be achieved by applying control of these variables, for example, less investment in inventory, better product quality, less waste, reduced storage space requirements, greater flexibility, increased productivity and improvement of the competitive position of the company (Hariga, 2000; Chuang *et al.*, 2004; Lin, 2009; Glock, 2012).

Periodic review (PR) policy provides solutions to problems of inventory management in many real-life situations. This policy indicates that the stock is completed at periodic intervals to reach a predefined level of inventory (Lau and Lau, 2003). Note that the materials are subject to a time-limited shelf life expiration date, which restricts but in turn gives a sufficiently long period for review and replacement. Periodic review for multiple products can include restrictions on storage volumes, maximum amount of orders per period and maximum availability of capital stocked inventory, among others (Lee and Lee, 2013).

Multiple products can be jointly replenished with the same cycle (Khouja and Goyal, 2008) by a grouping them into a fixed amount of product sets. To achieve grouping of products supplied jointly, it is possible to use cluster analysis, which groups individual by distances between them around different variables, forming homogeneous clusters which are heterogeneous between them. Because the technique assumes that the variables to group should have no correlation with each other, it is possible to first apply principal components or factor analysis (Pardo and Del Campo, 2007). For details about the cluster analysis, see Johnson and Wichern (2007).

Given the research gap existing in the characterization of the groups of products conformed to make JRP, an important contribution we want to make in this paper is to include various quantitative variables such as: expiration time, storage volumes, unit costs of purchase, level of urgency of use of material, among others, through the use of cluster analysis as a method of grouping of items in JRP.

*2.3 Stochastic programming*

As its name suggests, the stochastic programming problems is mathematical programming (linear, nonlinear, integer, etc.) whose formulation appears some stochastic element whose value is unknown but can estimate its probability distribution (Shapiro and Dentcheva, 2014). The expected value of the objective function based on the inventory TC must be optimized in PR policy for JRP of an assortment of products. Stochastic programming can be used to solve this optimization problem by the differential evolution (DE) algorithm, which belongs to the family of genetic algorithms, mimicking the process of natural selection in an evolutionary manner; for details about the DE algorithm, Price *et al.* (2006), Thangaraj *et al.* (2010) and Wanke and Leiva (2015).

*2.4 Problem to solve through of the inventory policy propose*

In this work, we propose a methodology based on inventory models for JRP for multiple products grouped by several variables, for supply from a single distributor with DGS. In application, it is carried out by cluster analysis to identify homogeneous products considering multi variables that can serve to characterize the groups to form for supply an assortment of products. We consider the DPUT of each product is an RV following a probability distribution, which has dependence over time being described by GARMA models. Optimization of the inventory TC is carried out by stochastic programming. We apply the proposed methodology to a pharmaceutical study, where drugs are segmented forming homogeneous clusters, but heterogeneous between them. Segmentation is used to determine unique replenishment periods in each group of drugs, according to the characteristics and appropriate periods of the members of each group supply from a single distributor with DGS.

The benefits of inventory models are evaluated by efficiency indicators. Some of them are financial, such as TCs type inventory or sales relative rotation of stocks available; while others are operational, such as the ability to meet demand with inventory planned by the inventory model (called" fill-rate"), or the expected shortage per cycle. In this paper will deal TCs as financial indicator of an assortment of products (Wanke *et al.*, 2016).

## 3. Methodology

*3.1 Demand during lead-time plus review period*

Let $Y_t$ be an RV corresponding to the DPUT at time $t$. In addition, let the RV $W$ be the cycle time ($R$) plus the LT ($L$) between the ordering of a product and its delivery, which has mean $\mathrm{E}(W) = \mathrm{E}(R) + \mathrm{E}(L) = \lambda_W$ and variance $\mathrm{Var}(W) = \sigma^2_W$. Furthermore, $W$ is independent from each element of the sequence of IID RVs $\{Y_t, t \in \mathbb{N}\}$, which has mean $\mathrm{E}(Y_t) = \mu_{Y_t}$ and variance $\mathrm{Var}(Y_t) = \sigma^2_{Y_t}$. Assume that orders do not cross (Hayya *et al.*, 2008). Moreover, let $D$ be the LTD plus review period (LTD+RP) for the product, which is the random sum given by:

$$D = Y_1 + Y_2 + \cdots + Y_W,$$

with probability density function (PDF) $f_D$ defined on [0, $\infty$) (non-negative support), cumulative distribution function (CDF):

$$F_D(d) = \int_0^d f_D(v)\,\mathrm{d}v$$

and quantile function (QF) $d(q) = F_D^{-1}(q)$, for $0 < q < 1$. The expectation and variance of $D$ are, respectively, expressed as:

$$\mathrm{E}(D) = (\mathrm{E}(R) + \mathrm{E}(L))\mathrm{E}(Y_t) = \mathrm{E}(W)\mathrm{E}(Y_t) = \lambda_W \mu_{Y_t}$$

and

$$\mathrm{Var}(D) = \mathrm{Var}(W)\mathrm{E}^2(Y_t) + \mathrm{E}(W)\mathrm{Var}(Y_t) = \mathrm{Var}(W)\mu^2_{Y_t} + \lambda_W \sigma^2_{Y_t}.$$

Note that, in general, $W$ and $Y_t$ can be modeled by any discrete or continuous distribution.

*3.2 GARMA model*

The assumption of IID RVs for DPUTs $Y_t$ depending over time $t$, for $t = 1, \ldots, T$, and for the LTD $D$ can be violated. GARMA models are derived from GLM, with the response variable belonging to the $\Upsilon$-parametric exponential family of distributions with PDF given by:

$$f_{Y_t}(y_t|\Upsilon_t) = \exp(y_t \Upsilon_t + a_t b(\Upsilon_t) + c_t),$$

where $b(\dot{})$ is a known function, $a_t$ and $c_t$ are sequences of constants. Information on past time $\Omega_{t\text{-}1}$ is summarized in the state variable $\eta_t$. The systemic component of model GARMA($p$, $q$) is described by a link function $g$ of the mean and standard deviation of data at time $t$ are calculated as:

$$\mu_{Y_t|\Upsilon_t} = \mathrm{E}(Y_t|\Upsilon_t) = \sum_{t=1}^{T} Y_t f_{Y_t}(y_t|\Upsilon_t),$$

and

$$\sigma_{Y_t|\Upsilon_t} = \sqrt{\left(\mu_{Y_t|\Upsilon_t} + \alpha \mu^2_{Y_t|\Upsilon_t}\right)},$$

respectively. In last expression $\alpha$ is a dispersion parameter in the usual GLM for the negative binomial type I distribution that will be busy in this paper. Then, $\eta_t = g(\mu_{Y_t|\Upsilon_t})$ corresponds to the mean of the variable of interest to be predicted conditional to past information $\Omega_{t\text{-}1}$ given by:

$$\eta_t = g(\mu_{Y_t|\Upsilon_t}) = \boldsymbol{x}_t^\top \boldsymbol{\beta} + \sum_{j=1}^{p} \phi_j \{g(y_{t-j}) - \boldsymbol{x}_{t-j}^\top \boldsymbol{\beta}\} + \sum_{j=1}^{q} \theta_j \{g(y_{t-j}) - \eta_{t-j}\}$$

where $\phi$ and $\theta$ corresponds to the ARMA components of a model of orders $p$ and $q$, respectively. Note that $\boldsymbol{\beta}_t^\top = (\beta_0, \beta_1, \ldots, \beta_n)$ is a vector of coefficients associated with $n$ covariates with dependence over time denoted by $\boldsymbol{x}_t^\top = (x_0, x_{1,t}, x_{2,t}, \ldots, x_{n,t})$, with $x_0 = 1$. The link function of the GARMA model generally is the identity or logarithmic function and the corresponding model variance is assumed to be constant over time (Benjamin *et al.*, 2003).

*3.3 Parameter estimation*

Given $n$ observations of $Y_t$, for $t = 1, \ldots, T$, the likelihood function is constructed as the product of conditional PDFs of $Y_t$ given $\Omega_{t\text{-}1}$. The state vector $\eta_t$ is a function of the

GARMA parameters $\phi^\top$, $\theta^\top$ and $\beta^\top$, where each time embodies these conditioning variables. Therefore, the corresponding log-likelihood function is given by:

$$\ell\left(\Upsilon^\top, \phi^\top, \boldsymbol{\theta}^\top, \boldsymbol{\beta}^\top\right) = \sum_{t=1}^{T} \log\left(f_{Y_t}\left(y_t | \boldsymbol{\eta}_t\left(\Upsilon^\top, \phi^\top, \boldsymbol{\theta}^\top, \boldsymbol{\beta}^\top\right)\right)\right). \quad (1)$$

By differentiating (1) for each of the parameters, it is possible to obtain their maximum likelihood (ML) estimates.

*3.4 Density forecast*

When formulating a statistical model, sample data are used to estimate its parameters. Then, we can employ the estimated model for diverse purposes. However, one has no idea about how good and/or reliable its predictions are out of this sample. Thus, one could evaluate the out-of-sample performance of the model by splitting this sample into two parts, one of them being a training set that could correspond to the first two thirds of the data. Hence, one can estimate the model parameters with the training set and then evaluate out-of-sample performance with the third remaining of the data. This is known as out-of-sample testing. If one estimates the model parameters and verifies the assumptions on which this model relies using the data sample with all of the thirds, then this is an in sample testing. Thus, out-of-sample testing should be considered in the model validation process. Out-of-sample predictive performance can be evaluated by the DF technique. Specifically, let $\{f_{Y_t}(y_t|\Upsilon_t)\}$ denote a sequence of one-step-ahead DFs engendered by a model and $\{p_{Y_t}(y_t|\Upsilon_t)\}$ denote for a sequence of PDFs defining the data generating process. In the DF technique, it must be checked whether:

$$\{f_{Y_t}(y_t|\Upsilon_t)\} = \{p_{Y_t}(y_t|\Upsilon_t)\} \quad (2)$$

or not. However, to test the truth of (2) is not an easy task, because $\{p_{Y_t}(y_t|\Upsilon_t)\}$ is not observed. We can use the probability integral transform given by:

$$z_t = \int_{-\infty}^{y_t} f_{Y_t}(u|\Upsilon_t)\mathrm{d}u \quad (3)$$

to carry out the mentioned task. Diebold *et al.* (1998) demonstrated that, under a null hypothesis based on (2), the sequence of probability transforms $\{z_t\}$ of $\{y_t\}$ with respect to $\{f_{Y_t}(y_t|\Upsilon_t)\}$, given in (3), is generated from IID U (0,1) RVs. Thus, first, one can consider a graphical analysis by using the histogram and autocorrelation function (ACF) and/or partial ACF (PACF) plots of the sequence $\{z_t\}$ to detect departures from the IID U (0,1) assumption. Then, the Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) tests and the Ljung-Box (LB) test can be used to corroborate goodness of fit and independence, respectively, detected in the graphical analysis of $\{z_t\}$. This indicates to us that the empirical sequence of probability integral transforms implied by the model given in (3) can be used for testing the hypothesis based on (2) and, consequently, the out-of-sample performance of this model. We propose Algorithm 1 for out-of-sample testing, which summarizes the steps of the DF technique.

**Algorithm 1** DF technique

1: Consider a sample of $n$ observations denoted by the sequence $\{y_t\}$;

2: Take the first two thirds of the sample defined by $\{y_t\}$ in Step 1 of

Algorithm 1, obtaining a training set of size $2n/3$ denoted by the sequence $\{y_t'\}$, and estimate the model parameters with $\{y_t'\}$;

3: Estimate now the corresponding PDF $f_{Y_t}(u|\Upsilon_t)$ defined in (3) associated with the model using the results obtained in Step 2 of Algorithm 1;

4: Forecast the sequence $\{z_t\}$ defined in (3) with the PDF estimated in Step 3 of Algorithm 1 and the last third of the sample, corresponding to a set of size n/3 denoted by the sequence $\{y_t''\}$;

5: Check uniformity of the sequence $\{z_t\}$ forecasted with $\{y_t''\}$ obtained in Step 4 of Algorithm 1 first by its histogram and then corroborate it by the AD and KS tests;

6: Prove independence of the sequence $\{z_t\}$ forecasted in Step 4 of Algorithm 1 by its ACF and/or PACF plots and then corroborate it by the LB test;

7: Use the uniformity and independence of the sequence $\{z_t\}$ forecasted in Step 4, verified in Steps 5 and 6 of Algorithm 1, to evaluate the out-of-sample performance of the model;

8: Employ the result obtained in Step 7 of Algorithm 1 to compare the performance of a model in relation to other.

Forecasts using GARMA models may be carried out in an analogous way to that employed for generalized autoregressive with conditional heteroscedasticity models; Tsay (2009). Thus, based on GARMA models and supposing that the forecast origin is $j = m$ and its horizon is $h$, we have the $h$-step ahead forecast is obtained from $y_{m+h}$, with initial prediction value $y_m$ at the origin $m$ and forecast error $e_m(h) = y_{m+h} - y_m(h)$, for $h \geq 1$.

### *3.5 Inventory model periodic review in joint replenishment problem of group of product using multivariate cluster analysis*

We will compare cost savings obtained for an individual and joint replenishment for a set of products, using an inventory model of RP. This inventory system checks inventory position periodically and reorder the LTD plus review period. The optimal values for the cycle time ($R$) and the maximum inventory level (which is a function of safety stock factor, $k_p$) should be determined to minimize joint cost function of order, storage and shortage. However, some functions and demand distributions make difficult to solve the inventory problem analytically. Given the convenience and less complexity in the resolution offered by the heuristic approaches, which correspond to practical or informal techniques or procedures to solve problems, it is that they enjoy a great use in operations research for a long time (Simon and Newell, 1958). As a result, approximations and heuristic are often used in periodic review inventory system. We assume normality for LTD plus the review period.

The objective is to develop an effective heuristic to solve a multi-item periodic review inventory in JRP with a review period constraint under stochastic demand.

To form clusters of products with homogeneous behavior intra groups and heterogeneous behavior inter groups for several grouping variables, it is necessary to evaluate the correlation between these variables. If correlations exist, it is necessary to reduce dimensionality by factor analysis and form new uncorrelated grouping variables. The extraction method of variance with varimax rotation ensures that each extracted factor possesses maximum shared variance with the original variables that composes it. The criteria to carry out the reduction of dimensionality are: themselves >1 for each factor, the sum must collect inertia values above 70 per cent, confirming the statistical significance of the factor loadings with magnitude greater than 0.7, and discarding loads <0.5. With the factor scores for each product, we proceed to conduct the cluster analysis. First, a hierarchical clustering of standardized factor scores of each product can be performed by linking the Ward algorithm, which minimizes the intra-group variance and finds a dendrogram finds with many preliminary groups shape. Knowing that, at a smaller distance, clusters are more homogeneous, the process is stopped when the horizontal lines are very long. Members and characteristics of each hierarchical clustering preliminary factorial scores are found occupying no hierarchical cluster analysis in k-means, which must be confirmed by an variance analysis.

### *3.6 Stochastic programming*

The problem of stochastic programming to be formulated should consist of:

- decision variables involved in each model administration inventory associated with the product to optimize; and
- maximize the objective function corresponding to TCs.

The TC of the product $i$ is expressed as the sum of the costs (in monetary units):

- generation of purchase orders per order ($O_i$) per year;
- storage per unit of product ($H_i$) per year; and
- of shortage or rupture per unit of the component ($S_i$) a year.

It is assumed that demand is continuing every day of the year. The optimization problem of the expected inventory TC can be visualized as a model of stochastic programming formulated as $\min\{Z_j = C(R_j, k_{p_i})\}$, subject to a $R_j > 0$, where the group $j$ is a subset of products $i = 1, \ldots, n$. Here, $R_j$ is a review period for the group $j$, and $k_{p_i}$ is a secure factor of the product $i$, in a PR system for all products $i$ of the group $j$. The review period $R_j$ may require restrictions. For example, in supply of drugs in public hospitals, replenishment time cannot exceed the period of expiration of the products ($E_i$), then $R_j \leq E_i$. As supply is a single distributor, only the major ordering cost for a group of products ($O_j$) is considered, and is equal to a minor ordering cost of one single product ($O_i$) that not considered in supply of group of products. Other constraints are: $S_i > 0$, $H_i > 0$, $O_i > 0$, $\mu_i > 0$, where $\mu_i$ is the rate of DPUT of a product $i$ conditional on past information and $\sigma_i$ the standard deviation also conditional on past information.

### *3.7 Inventory model periodic review for multiple products*

The inventory model of PR for all products $i = 1, \ldots, n$ in the group $j$ in JRP minimizes the expected annual costs min $\{E(TC_i)\}$ = min $\{E(O_i + H_i + S_i)\}$. This sum of terms is described as:

$$\min\left\{\sum_{i=1}^{n} \mathrm{TC}_i\left(R_j, k_{p_i}, \max_i, S\left(\max_i\right)\right)\right\} = \min\left\{\frac{O_j}{R_j} + \sum_{i=1}^{n}\left(\frac{12\mu_i R_j}{2} + k_{p_i}\sigma_i\sqrt{R_j + l_i}\right)H_i\right\} + \min\left\{\sum_{i=1}^{n}\frac{S(\max_i)S_i}{R_j}\right\},$$

subject to $R_j$, $H_i$, $O_j$, $D_i$, $\mu_\mathrm{i} > 0$, and $R_j \leq E_i$, for all group $j$, where $k_{p_i}$ is the optimal standardized QF of the product $i$ associated with a service level of $p \times 100$ (or amount of standard deviations –SDs– of the LTD plus review period). Both $R_j$ as $k_{p_i}$ correspond to decision variables that minimize the inventory TC. For the members of the product $i$ of each group $j$, each element has a cycle unrestricted supply ($R_j$). $D_i$ corresponds to the LTD+RP for product $i$, while the function of shortage $S(\max_i)$, can be calculated as:

$$S(\max_i) = \int_{\max_i}^{mD_i} \left(D_i - \max_i\right) f_{D_i}(D_i)\, \mathrm{d}D_i,$$

where $mD_i$ corresponds the maximum LTD+RP, $f_{D_i}$ is the PDF of the LTD+RP and the maximum supply of product $i$ corresponds to:

$$\max_i = \left(R_j + l_i\right)\mu_i + k_i \sigma_{D_i}.$$

Note that for modeling the shortages caused by the LTD plus review period, we assume that the variable $D_i$ is normally distributed (Wanke and Leiva, 2015), because although data of demand for products have a discrete nature, usually, these are modeled by continuous distributions.

As is clear from the stochastic programming raised, it is required to establish the values of various parameters related to conditional probability functions ($\mu_i$, $\sigma_i$, $S(\max_i)$), whose obtain is explained below.

*3.8 Summary of the methodology*

Algorithm 2 summarizes our methodology in four main steps divided in 15 sub-steps based on the aspects detailed in Subsections 3.1 to 3.7, from the collection of data until the establishment of the TCs, and can be occupied to evaluate the results or performance of inventory management models in grouping for JRP with periodic review in relation to the individual periodic review of the products. We recall this algorithm considers the demand dependent to past information for products, although it can also be used to compare measures of performance of inventory models obtained for JRP groupings obtained considering the demand as non-time dependent (IID case). Once that all the products are considered, the total contribution of the products used in the service are considered in the optimization.

**Algorithm 2** Main methodological steps

1: Collect demand data for the product ($i = 1, \ldots, n$) in each month during 2 years.

2: For the statistical analysis:

2.1 Carry out an autocorrelation study for data collected in Step 1 examining plot of the ACF and PACF to detect possible dependence to past information.

2.2 Propose GARMA models considering negative binomial distribution for the response variable (which is a monthly count is sometimes zero), for each product, selecting those through the Akaike criteria was lower.

2.3 Run a check on behavior of residues of the fit of the proposed models, showing the ACF and PACF, density residuals and QQplot to show that they follow a normal distribution.

2.4 Check the reliability predictive models using histograms PIT.

2.5 To estimate the parameters of the PDF of the conditional

```
    demand past information.
    2.6 Find the mean and standard deviation conditional on past in-
    formation for each product.
    2.7 Before obtaining product groups, check the correlations of
    the grouping variables to occupy. In the event that these are cor-
    related, obtain uncorrelated transformations of the primitive
    variables by using the technique of principal components.
    2.8 Build the product cluster using a hierarchical clustering
    method under the criteria of Ward, considering the distances
    between the variables uncorrelated step 2.7.
    2.9 Corroborating the formation of clusters by k-means method,
    indicating the products that belong to each group.
3: For the inventory analysis:
    3.1 Select the suitable inventory model RP for all group of prod-
    ucts.
    3.2 Find the optimal inventory elements (R_j, k_{p_i}) based on condi-
    tional distributions established and parameters of Step 2.5 and
    2.6, assuming normality for demand during LT plus review period.
4: For the financial analysis:
    4.1 Compute the TC for the each group product of the optimal policy
    obtained in Step 3.2.
    4.2 Determine the corresponding the arguments (R_j, k_{p_i}) that maxi-
    mize inventory TC function, and shortage expected by cycle for
    each product.
    4.3 Repeat steps 4.1 and 4.2 for individual products, and compare.
```

## 4. Application

The drug supply in pharmacy units of Chilean primary health centers is channeled through their central warehouse, which acts as an intermediary between suppliers and output units (OU). The OUs receive the demand for drugs, including its own pharmacy, which performs dispensing of prescriptions to patients. This warehouse needs the storage, conservation and distribution of such drugs. Supply of warehouse is carried out by a supplier. The warehouse delivers the products on a monthly basis to all OUs by using aggregated demand requirements for each of them in the same period.

### *4.1 The data set*

To validate the proposed methodology, we use real-world monthly demand data for an assortment of 223 pharmaceutical products, which it has extracted a set of nine products auto correlated monthly demands used in this example. They are shipped from the warehouse and delivered to a family health center, located at the city of Concon, Chile, for a study of supply policy conducted by Fernando Rojas in the University of Valparaiso, Chile, during 24 months of the years 2014-2015 (from January 1 to December 31). The data set collected for each variable considers the following grouping of products by variables:

- generic product (only identification);
- expected value of the quantities demanded depending on the time;
- SD of the monthly demands;
- unit costs of purchase;

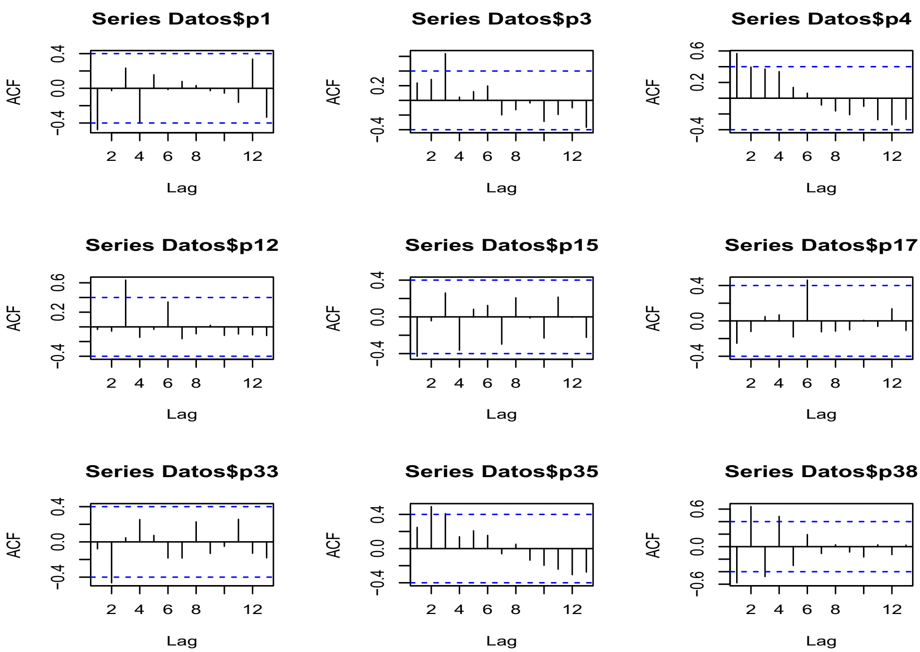
- pharmaceutical form (ordered according to the storage volume); and
- therapeutic use (ordered according to the urgency of use).

### 4.2 Computational framework

R is non-commercial and open source software for statistics and graphs, which can be obtained at no cost from www.r-project.org. The statistical software R is being currently very popular in the international scientific community. For a use of this software in inventory models, see Rojas *et al.* (2015). Some R packages related to statistical distributions that may be useful in inventory models are available at http://CRAN.R-project.org (Stasinopoulos and Rigby, 2007; Leiva *et al.*, 2008; Barros *et al.*, 2009). The expected value for monthly demand products is calculated by using GARMA models implemented in the R software by the packages gamlss.util () for time series and gamlss () for different probability distributions. To build groups of drugs, we carry out factor and cluster analyses by variables, using an R package named cluster, while the stochastic programming of inventory models is performed by the packaged DEoptim () of the same software.

### 4.3 Data analysis

In the first instance, we show that the assumptions of temporal dependence of the demand are fulfilled, and that this random variable can be modeled plausibly by a GARMA model. The above is the basis of the proposed methodology. Figures 1 and 2 show the ACF and




**Source:** Authors

**Figure 1.** ACF plots of DPUTs for illustrative data set

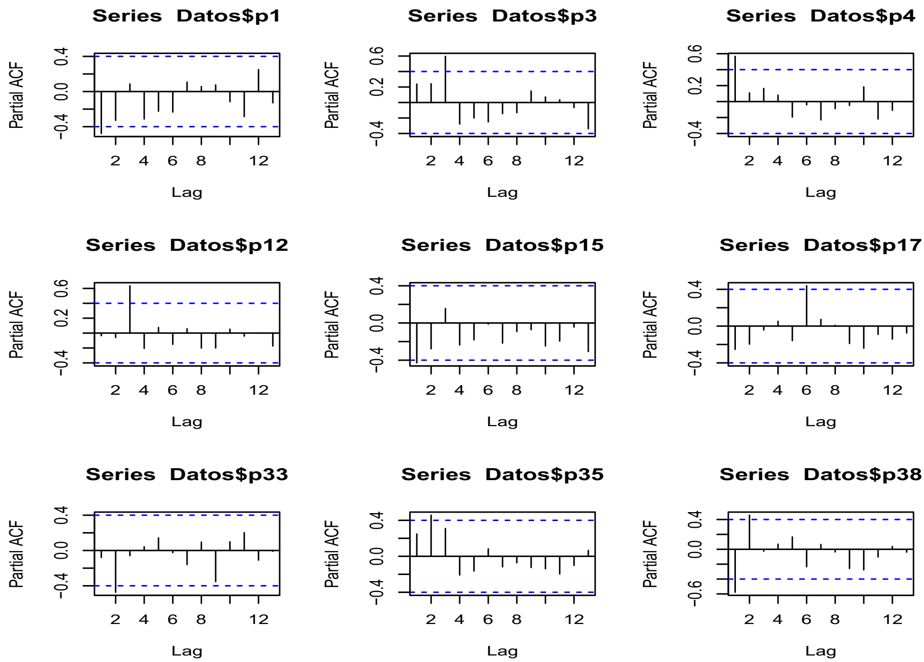



**Figure 2.**
ACF plots of DPUTs for illustrative data set

**Source:** Authors

PACF plots of DPUTs of an illustrative set of products. Note that the examination of these figures shows that the probability distributions of these variables are not IID.

Table I shows an descriptive statistics and Table II shows an illustrative example for a set of products with the best fit to GARMA models with parameters of the negative binomial distribution chosen by the Akaike information criteria (AIC). AIC is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection. In this table are displayed: mean ($\mu_{Y_{t_i}|\Upsilon_t}$),

**Table I.**
Descriptive statistic of monthly demand of set of products

| Product | Mean | Sd | IQR | 0% | 25% | 50% | 75% | 100% | *n* |
|---|---|---|---|---|---|---|---|---|---|
| p1 | 29458,33 | 20580,65 | 35000 | 0 | 10000 | 28000 | 45000 | 66000 | 24 |
| p2 | 2626,25 | 1991,53 | 2500 | 0 | 1500 | 2010 | 4000 | 6700 | 24 |
| p3 | 1447,08 | 1798,83 | 2302,5 | 0 | 0 | 0 | 2302,5 | 5000 | 24 |
| p4 | 376,25 | 823,65 | 7,5 | 0 | 0 | 0 | 7,5 | 3000 | 24 |
| p5 | 291,67 | 464,31 | 1000 | 0 | 0 | 0 | 1000 | 1000 | 24 |
| p6 | 479,17 | 950,05 | 500 | 0 | 0 | 0 | 500 | 4000 | 24 |
| p7 | 295,83 | 130,15 | 200 | 0 | 200 | 300 | 400 | 500 | 24 |
| p8 | 24,17 | 23,89 | 40 | 0 | 0 | 17,5 | 40 | 80 | 24 |
| p9 | 2128,33 | 2089,89 | 3202,5 | 0 | 0 | 2000 | 3202,5 | 7000 | 24 |

**Source:** Authors

**Table II.** Order of GARMA model for DPUTs of illustrative product

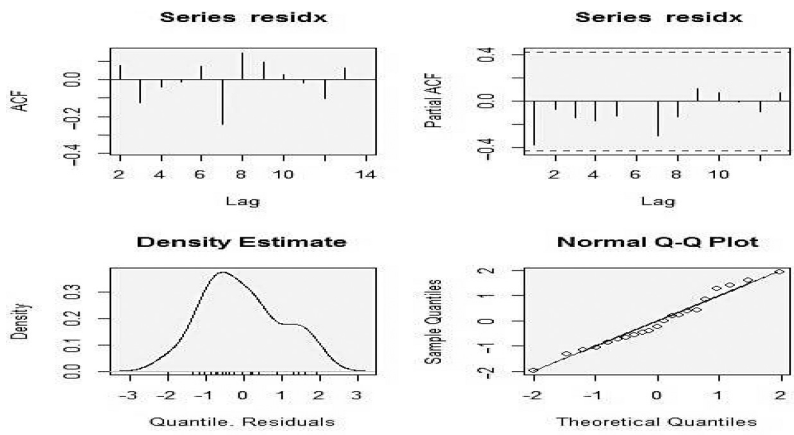
| Product $i$ | $\mu_i \mid \Upsilon_t$ (units/ month) | $\sigma_i \mid \Upsilon_t$ (units/ month) | $\alpha_i$ | ARMA ($p$, $q$) | AR1 (SE) | AR2 (SE) | MA1 (SE) | M A2 (SE) | Intercept (SE) | AIC |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11904 | 11053 | 0,86 | 2,1 | −0,41 (0,39) | −,05 (0,07) | 0,30 (0,27) | – | 10,21 (0,23) | 485,2 |
| 2 | 1116 | 757 | 0,46 | 1,2 | 0,36 (0,31) | – | −0,33 (0,32) | 0,09 (0,09) | 7,98 (0,46) | 371,0 |
| 3 | 639 | 279 | 0,19 | 2,2 | 1,0 (0,36) | −0,04 (0,36) | −0,75 (0,34) | 0,41 (0,24) | 21,47(54,57) | 209,0 |
| 4 | 140 | 45,25 | 0,09 | 2,2 | 1,0 (NA) | −1,5e − 3 (NA) | −0,89 (0,24) | 0,44 (0,16) | 1384 (752) | 120,8 |
| 5 | 2,62 | 1,77 | 0,07 | 2,2 | −1,0 (0,75) | −0,21 (0,28) | 0,14 (0,13) | 0,71 (0,24) | 1,94 (0,25) | 139,3 |
| 6 | 1,33 | 1,21 | 0,07 | 2,2 | −0,79 (0,16) | 0,005 (0,16) | −0,74 (0,03) | −1,0 (0,02) | −0,28 (0,04) | 175,5 |
| 7 | 190 | 272 | 2,04 | 2,2 | −0,26 (0,66) | −0,02 (0,36) | 0,29 (0,69) | 0,019 (0,41) | 5,67 (0,18) | 287,2 |
| 8 | 16 | 21 | 12,419 | 2,2 | 0,1 (0,58) | 0,15 (0,55) | 0,16 (0,62) | 0,29 (0,48) | 3,59 (0,41) | 168,7 |
| 9 | 4,41 | 2,93 | 0,21 | 1,1 | 0,58 (0,28) | – | 0,49 (0,30) | – | 7,38 (0,45) | 327,3 |

**Source:** Authors

SD ($\sigma_{Y_{t_i}|\Upsilon_t}$) and positive dispersion parameter ($\alpha_i$) of the distribution of DPUT, order ARMA, autoregressive (AR), moving average (MA) and intercept coefficients with standard error (SE) respective, and value of AIC. The models exclude explanatory variables.

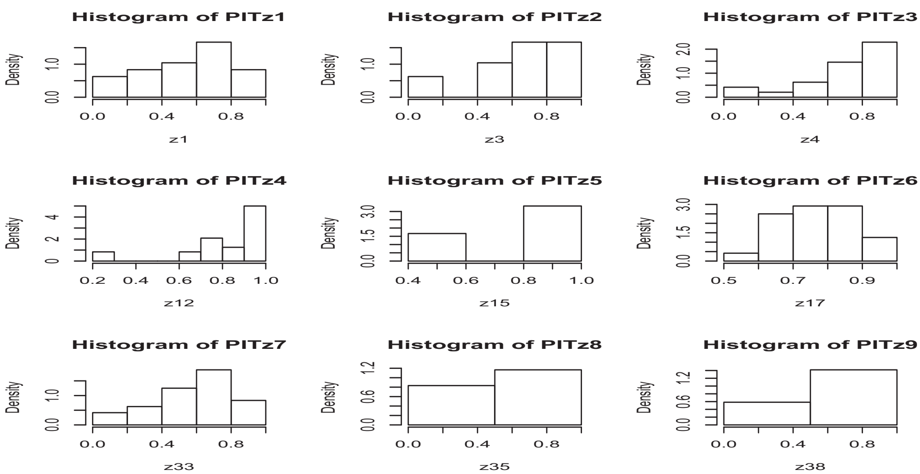
To confirm the correct fit of the proposed GARMA model are examined four plots for checking the normalized (randomized for a discrete response distribution) quantile residuals of a fitted GARMA object (GarmaFit), referred to as residuals below: a plot of the ACF and PACF of the residuals, a kernel density estimate of the residuals and a normal quantile versus quantile plot of the residuals; see example for product 1 in Figure 3.

Figure 4 shows histogram of probability integral transformed of an illustrative set of products. The examination of these histograms shows reasonable uniform distributions [0-1]




**Source:** Authors

**Figure 3.**
Confirmatory analysis GARMA model proposed fit to an illustrative product



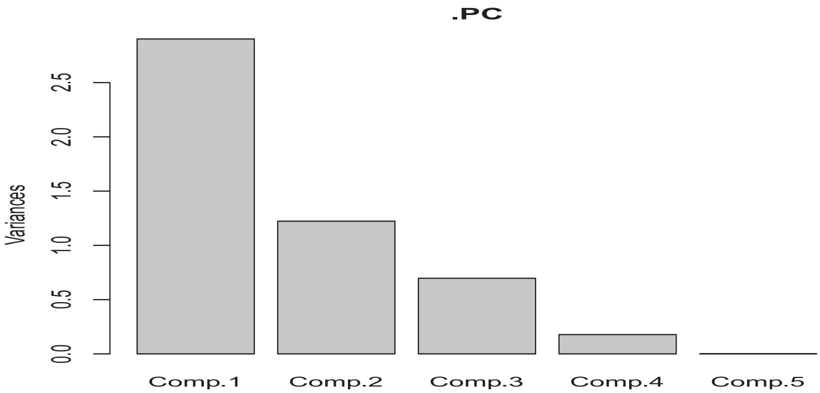

**Source:** Authors

**Figure 4.**
Histogram of probability integral transformed of an illustrative products

and absence of autocorrelation of probability integral transformations of forecasts of the GARMA models proposed for products, indicating a good evaluation for all forecast models.

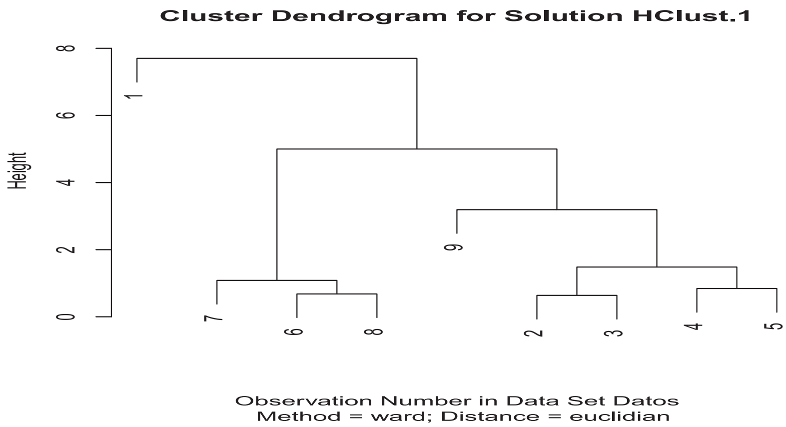
To determine the factors non-correlated to occupy as grouping variables in the cluster analysis used the technique of dimensional reduction of principal components. This technique is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables (entities each of which takes on various numerical values) into a set of values of linearly uncorrelated variables called principal components. Figure 5 shows that the extraction of two components is capable of collecting the greater amount of the variance of the primitive variables of grouping.

Figure 6 show the dendogram of groupings of products formed from scoring of principal components non-correlated realized by method hierarchical of Ward. Note that the hierarchical clustering and subsequently corroborated by the k-means method results in the




**Source:** Authors

**Figure 5.** Variance extracted by principal components




**Source:** Authors

**Figure 6.** Dendogram of groupings of products

formation of four groups of products. Table III and IV show a summary of policy for PR of products grouped and individual products, where order cost is US$77,78/order, holding cost is US$0,26/unit/year and penalty cost for shortage is 0,05 USD/unit/cycle. Note that the TC for grouped products are 6 per cent lower than to individual policy and shortages units expected decrease significantly in grouped policy. In both the cases, the DPUT was described by GARMA model.

To compare our results when we model the DPUT by means of GARMA model, regarding the non-consideration of the serially distributed DPUT over time, that is, considering this random variable as an IID case and assuming normality as this variable has been considered, we replicate the results of the Figures 5 and 6, as well as the Table III, but now in the case IID $\sim N$ (mean, standard deviation) for each item 1,. . .,9. Note that, the performance of TC obtained by our proposal to model DPUT not IID with GARMA model is it is better to apply the same clustering algorithm but considering the DPUT $\sim N$ (mean, standard deviation) for each item 1,. . .,9. In this case, to determine the factors non-correlated to occupy as grouping variables in the cluster analysis used the technique of dimensional reduction of principal components. The Figure 7 show that the extraction of only one component is capable of collecting the greater amount of the variance of the primitive variables of grouping.

Figure 8 show the dendogram of groupings of products formed from scoring of principal components non-correlated realized by method hierarchical of Ward. Note that the hierarchical clustering and subsequently corroborated by the k-means method results in the formation of four groups of products, different from the case of DPUT described by GARMA model, where both the average and standard deviation parameters are generally

| Groups | Products | Replenishment time (years) | Safety factors for products | Shortage for products (un/cycle) | TC (USD) |
|---|---|---|---|---|---|
| 1 | 1 | 0,07 | 0,35 | 767,04 | 3240,22 |
| 2 | 2/3/4/5 | 0,18 | 0/0/0/0 | 466,14/414,78/ 127,74/0,71 | 1136,23 |
| 3 | 6,7,8 | 0,495 | 0/0/0 | 0,56/37,85/6,4 | 321,65 |
| 4 | 9 | 0,08 | 0,18 | 352,81 | 2250,7 |

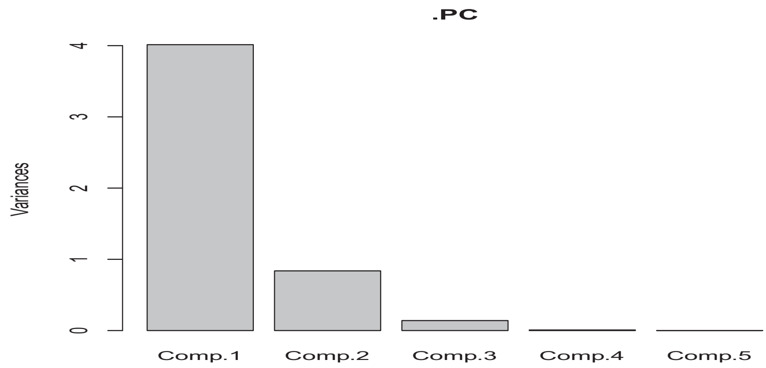
**Source:** Authors

**Table III.** Summary of optimal policy for PR of products grouped

| Products | Review period (years) | Safety factors for products | Shortage for products (un/cycle) | TC (USD) |
|---|---|---|---|---|
| 1 | 0,07 | 0,35 | 767,04 | 3240,22 |
| 2 | 0,22 | 0 | 2106 | 808,1 |
| 3 | 0,29 | 0 | 2408 | 612,09 |
| 4 | 0,61 | 0 | 3467 | 272,66 |
| 5 | 2 | 0 | 6248 | 41,0 |
| 6 | 2 | 0 | 6248 | 41,0 |
| 7 | 1,79 | 0 | 5913 | 89,73 |
| 8 | 2 | 0 | 6248 | 43,37 |
| 9 | 0,08 | 0,18 | 352,81 | 2250,7 |

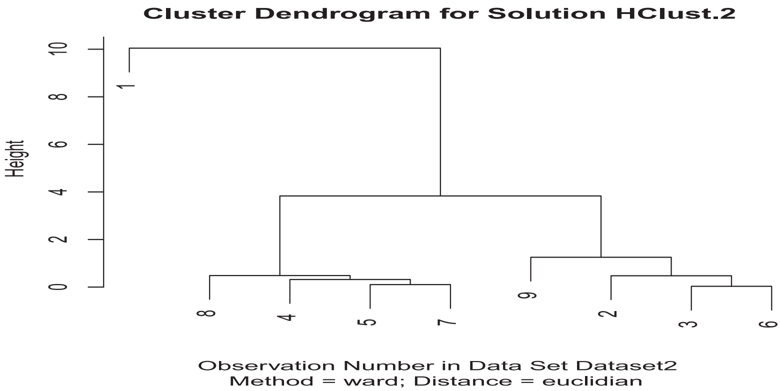
**Source:** Authors

**Table IV.** Summary of optimal policy for PR of individual products




**Source:** Authors

**Figure 7.** Variance extracted by principal components case IID ∼ *N* (mean, standard deviation)




**Source:** Authors

**Figure 8.** Dendogram of groupings of products case IID ∼ *N*(mean, standard deviation)

lower. Table V show a summary of policy for PR of products grouped considering the DPUT ∼ *N* (mean, standard deviation) for each item 1, ,9, where order cost, holding cost and penalty cost for shortage are the same as in the case of DPUT described with GARMA model. Note that the TC for grouped products considering DPUT IID are 26 per cent larger than considering DPUT described by GARMA model.

| Groups | Products | Replenishment time (years) | Safety factors for products | Shortage for products (un/cycle) | TC (USD) |
|---|---|---|---|---|---|
| 1 | 1 | 0,07 | 0,35 | 767,04 | 3240,22 |
| 2 | 4/5/7/8 | 0,23 | 0/0/0/0 | 533,28/524,32/157,87/1,3 | 2836,23 |
| 3 | 2,3,6 | 0,63 | 0/0/0 | 1,63/48,74/12,5 | 428.34 |
| 4 | 9 | 0,08 | 0,18 | 352,81 | 2250,7 |

**Source:** Authors

**Table V.** Summary of optimal policy for PR of products grouped case IID ∼ *N* (mean, standard deviation)

## 5. Discussion

The policy inventories raised show an interesting way to achieve significant savings in TC and shortage expected for multi-product systems. The grouping of products that achieves multivariate cluster analysis considers multiple variables that influence the characterization of supply. The proposed supply cycle for groups of products is useful in institutions that have a supplier to meet your requirements, and that generally are characterized by high bureaucracy in your administrative systems as public hospitals, where the operation of a supply system as proposed would be facilitated (Bennett and Gilson, 2001; Birkett *et al.*, 2001)). The parameterization of the mean of a PDF under a temporal structure of a conditional autoregressive model to information passed as GARMA, achieves an efficient and dynamic characterization of a myopic policy involving the DPUT inventories and LTD plus review period. Although in this work we occupy a normal-type statistical distribution to model LTD plus review period, the proposal is valid for any statistical distribution that can be parameterized with respect to the mean in a time series model. This condition opens multiple areas of research in the area. In the future, it is possible to obtain improvements in the modeling raised occupying other than normal distributions, and even considering that in many cycles demand component is zero, requiring demand model using probability distributions for continuous and/or discrete data zero-inflated (Leiva *et al.*, 2016). The possibility to perform forecasts based on a conditional autoregressive model whose quality could be evaluated by the probability integral transformed, would update in $n$-steps ahead the myopic policy inventories.

## References

Atkins, D. and Iyogun, P. (1988a), “A heuristic with lower bound performance guarantee for the multi-product dynamic lot-size problem”, *IIE Transactions*, Vol. 20 No. 4, pp. 369-373.

Atkins, D.R. and Iyogun, P.O. (1988b), “Periodic versus can-order policies for coordinated multi-item inventory systems”, *Management Science*, Vol. 34 No. 6, pp. 791-796.

Balintfy, J.L. (1964), “On a basic class of multi-item inventory problems”, *Management Science*, Vol. 10 No. 2, pp. 287-297.

Barros, M., Paula, G. and Leiva, V. (2009), “An R implementation for generalized Birnbaum-Saunders distributions”, *Computational Statistics and Data Analysis*, Vol. 53 No. 4, pp. 1511-1528.

Bauwens, L., Giot, P., Joachim, G. and David, V. (2004), “A comparison of financial duration models via density forecasts”, *International Journal of Forecasting*, Vol. 20 No. 4, pp. 589-609.

Benjamin, M.A., Rigby, R.A. and Stasinopoulos, D.M. (2003), “Generalized autoregressive moving average models”, *Journal of the American Statistical Association*, Vol. 98 No. 461, pp. 214-223.

Bennett, S. and Gilson, L. (2001), *Health Financing: Designing and Implementing Pro-Poor Policies*, DFID Health Systems Resource Centre.

Birkett, D.J., Mitchell, A.S. and McManus, P. (2001), “A cost-effectiveness approach to drug subsidy and pricing in Australia”, *Health Affairs*, Vol. 20 No. 3, pp. 104-114.

Braglia, M., Castellano, D. and Frosolini, M. (2016), “Joint-replenishment problem under stochastic demands with backorders-lost sales mixtures, controllable lead times, and investment to reduce the major ordering cost”, *Journal of the Operational Research Society*, Vol. 67 No. 8, pp. 1108-1120.

Calfa, B.A. (2015), “Data analytics methods for enterprise-wide optimization under uncertainty”, Doctoral dissertation, Carnegie Mellon University, Pittsburgh.

Chuang, B.-R., Ouyang, L.-Y. and Chuang, K.-W. (2004), “A note on periodic review inventory model with controllable setup cost and lead time”, *Computers and Operations Research*, Vol. 31 No. 4, pp. 549-561.

Diebold, F.X., Gunther, T.A. and Tay, A.S. (1998), "Evaluating density forecasts with applications to financial risk management", *International Economic Review*, Vol. 39 No. 4, pp. 863-883.

Diebold, F.X., Hahn, J. and Tay, A. (1999), "Real-time multivariate density forecast evaluation and calibration: monitoring the risk of high-frequency returns on foreign exchange", *Review of Economics and Statistics*, Vol. 81 No. 4, pp. 661-673.

Dunsmuir, W.T. and Scott, D.J. (2015), "The glarma package for observation driven time series regression of counts", *Journal of Statistical Software*, Vol. 67 No. 7, pp. 1-36.

Eynan, A. and Kropp, D.H. (1998), "Periodic review and joint replenishment in stochastic demand environments", *IIE Transactions*, Vol. 30 No. 11, pp. 1025-1033.

Gilbert, K. (2005), "An arima supply chain model", *Management Science*, Vol. 51 No. 2, pp. 305-310.

Glock, C.H. (2012), "Lead time reduction strategies in a single-vendor–single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand", *International Journal of Production Economics*, Vol. 136 No. 1, pp. 37-44.

Hariga, M. (2000), "Setup cost reduction in (q, r) policy with lot size, setup time and lead-time interactions", *Journal of the Operational Research Society*, Vol. 51 No. 11, pp. 1340-1345.

Hayya, J., Bagchi, U., Kim, J. and Sun, D. (2008), "On static stochastic order crossover", *International Journal of Production Economics*, Vol. 114 No. 1, pp. 404-413.

Hillier, F. and Lieberman, G. (2005), *Introduction to Operational Research*, McGraw Hill, New York, NY.

Johnson, R.A. and Wichern, D.W. (2007), *Applied Multivariate Statistical Analysis*, Prentice-Hall, New Jersey.

Khouja, M. and Goyal, S. (2008), "A review of the joint replenishment problem literature: 1989-2005", *European Journal of Operational Research*, Vol. 186 No. 1, pp. 1-16.

Kristianto, Y., Gunasekaran, A., Helo, P. and Sandhu, M. (2012), "A decision support system for integrating manufacturing and product design into the reconfiguration of the supply chain networks", *Decision Support Systems*, Vol. 52 No. 4, pp. 790-801.

Lau, H. and Lau, A. (2003), "Nonrobustness of the normal approximation of lead-time demand in a (Q, R) system", *Naval Research Logistics*, Vol. 50 No. 2, pp. 149-166.

Lee, M.H. and Lee, D.J. (2013), "A multi-item stochastic demand periodic review system with a budget constraint", *Journal of Next Generation Information Technology*, Vol. 4, pp. 403-411.

Leiva, V., Hernandez, H. and Sanhueza, A. (2008), "An R package for a general class of inverse Gaussian distributions", *Journal of Statistical Software*, Vol. 26 No. 4, pp. 1-16.

Leiva, V., Santos-Neto, M., Cysneiros, F.J.A. and Barros, M. (2016), "A methodology for stochastic inventory models based on a zero-adjusted Birnbaum-Saunders distribution", *Applied Stochastic Models in Business and Industry*, Vol. 32 No. 1, pp. 74-89.

Lin, Y.-J. (2009), "An integrated vendor–buyer inventory model with backorder price discount and effective investment to reduce ordering cost", *Computers and Industrial Engineering*, Vol. 56 No. 4, pp. 1597-1606.

Namit, K. and Chen, J. (1999), "Solutions to the inventory model for gamma lead-time demand", *International Journal of Physical Distribution and Logistics Management*, Vol. 29 No. 2, pp. 138-154.

Pardo, C.E. and Del Campo, P.C. (2007), "Combinación de métodos factoriales y de análisis de conglomerados en R: el paquete FactoClass", *Revista Colombiana de Estadística*, Vol. 30, pp. 231-245.

Pasandideh, S.H.R., Niaki, S.T.A. and Ahmadi, P. (2018), "Vendor-managed inventory in the joint replenishment problem of a multi-product single-supplier multiple-retailer supply chain: a teacher-learner-based optimization algorithm", *Journal of Modelling in Management*, Vol. 13 No. 1, pp. 156-178.

Prabakaran, K., Sivapragasam, C., Jeevapriya, C. and Narmatha, A. (2013), "Forecasting cultivated areas and production of wheat in India using Arima model", *Golden Research Thoughts*, Vol. 3, pp. 2231-4063.

Price, K.V., Storn, R.M. and Lampinen, J.A. (2006), *Differential Evolution: A Practical Approach to Global Optimization*, Springer, Berlin.

Rojas, F. and Leiva, V. (2016), "Inventory management in food companies with statistically dependent demand", *Academia Revista Latinoamericana de Administración*, Vol. 29 No. 4, pp. 450-485.

Rojas, F., Leiva, V., Wanke, P. and Marchant, C. (2015), "Optimization of contribution margins in food services by modeling independent component demand", *Revista Colombiana de Estadística*, Vol. 38 No. 1, pp. 1-30.

Rojas, F., Leiva, V., Wanke, P., Lillo, C. and Pascual, J. (2019), "Modeling lot-size with time-dependent demand based on stochastic programming and case study of drug supply in Chile", *Plos One*, Vol. 14 No. 3, p. e0212768.

Shapiro, A. and Dentcheva, D. (2014), *Lectures on Stochastic Programming: Modeling and Theory*, Vol. 16, SIAM, Philadelphia.

Silver, E.A. (1973), "A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment", *Prod. Inventory Manage*, Vol. 2, pp. 64-74.

Simon, H.A. and Newell, A. (1958), "Heuristic problem solving: the next advance in operations research", *Operations Research*, Vol. 6 No. 1, pp. 1-10.

Stasinopoulos, D. and Rigby, R. (2007), "Generalized additive models for location, scale and shape (GAMLSS)", *Journal of Statistical Software*, Vol. 23 No. 7, pp. 1-46.

Thangaraj, R., Pant, M., Bouvry, P. and Abraham, A. (2010), "Solving multi objective stochastic programming problems using differential evolution", in Panigrahi, B., Das, S., Suganthan, P. and Dash, S., (Eds), *Swarm, Evolutionary, and Memetic Computing*, Vol. 6466, Springer, Berlin-Heidelberg, pp. 54-61.

Tsai, C.-Y., Tsai, C.-Y. and Huang, P.-W. (2009), "An association clustering algorithm for can-order policies in the joint replenishment problem", *International Journal of Production Economics*, Vol. 117 No. 1, pp. 30-41.

Tsay, R. (2009), "Autoregressive conditional duration models", in Mills, T.C. and Patterson, K., (Eds), *Handbook of Econometrics*, Vol. 2, Palgrave MacMillan, London, pp. 1004-1024.

Van Eijs, M.J.G., Heuts, R.M.J. and Kleijnen, J.P.C. (1992), "Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs", *European Journal of Operational Research*, Vol. 59 No. 3, pp. 405-412.

Wanke, P. and Leiva, V. (2015), "Exploring the potential use of the Birnbaum-Saunders distribution in inventory management", *Mathematical Problems in Engineering*, Vol. 827246, pp. 1-9.

Wanke, P., Ebwank, H., Leiva, V. and Rojas, F. (2016), "Inventory management for new products with triangularly distributed demand and lead-time", *Computers and Operations Research*, Vol. 69, pp. 97-108.

**Corresponding author**
Fernando Rojas can be contacted at: fernando.rojas@uv.cl